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Improving Hospital Efficiency and Economic Performance: A DEA Approach with Undesirable Factors in Tehran Emergency Wards

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ABSTRACT

Since the 1980s, Data Envelopment Analysis (DEA) has undergone remarkable advancements in both theoretical foundations and practical applications, surpassing initial expectations in the field. To optimize organizational performance, identifying and incorporating undesirable inputs and outputs is vital for improving system efficiency, minimizing waste, and enhancing resource allocation—ultimately contributing to economic efficiency. This research employs established DEA models to assess decision-making units' (DMUs) performance while explicitly accounting for undesirable factors. The results demonstrate that including undesirable inputs and outputs significantly influences the identification of the efficiency frontier, thereby affecting the comparative assessment of DMUs and providing a more accurate reflection of real-world economic constraints. Consequently, DMU efficiency and performance can be improved by reducing undesirable outputs and increasing desirable ones, pushing them toward the efficient frontier. In healthcare, this leads to better patient outcomes and more effective resource utilization. Economically, it translates to lower operational costs, improved resource allocation, and greater overall productivity in the healthcare system. To validate the proposed models, a case study was conducted using real-world data from 30 emergency wards in Tehran hospitals, comprising five desirable inputs, one undesirable input, four desirable outputs, and one undesirable output.

1. Introduction

Healthcare systems are characterized by complex and diverse structures, encompassing a wide range of care that includes primary, secondary, and post-acute services. Hospitals act as central organizational hubs within this intricate framework, delivering a wide range of diagnostic, therapeutic, and rehabilitative services [1]. Emergency wards of hospitals are an essential segment of hospitals, which are often overcrowded and overwhelmed, acting as the initial entry point for patients in need of immediate medical care. These units operate 24/7, serving a varied patient

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demographic that includes individuals facing life-threatening emergencies as well as those in search of treatment for less urgent issues [2]. The substantial number of patients receiving emergency care, over 33 million in just one country, highlights the globally faced issue of overcrowding and resource shortage within these crucial units. This ongoing challenge has been intensified by the unseen global health emergency caused by the COVID-19 pandemic [3]. The pandemic not only brought into light the existing inefficiencies but also underscored the dire need for effective disaster preparedness strategies, including the proactive identification of hospitals with limited surge capacity and insufficient readiness [4]. Consequently, improving the efficiency and effectiveness of healthcare systems has become a critical priority for developed nations aiming to ensure equitable access to high-quality care while also managing rising costs [5].

To address these challenges, Data Envelopment Analysis (DEA) has gained noticeable attention as a robust and flexible approach for assessing the effectiveness of healthcare systems and pointing out areas in need of improvement [6]. The fundamental capability of DEA to assess the relative efficiency of various decision-making units (DMUs), including hospitals or specific wards within them, renders it particularly vital in situations characterized by limited resources and the need for data-informed decision-making during and following the pandemic. Moreover, DEA can offer detailed insights into efficiencies specific to different types, allowing individual hospitals to pinpoint particular improvement opportunities and aiding in the creation of case-mix indices that facilitate meaningful comparisons among diverse hospitals serving different patient demographics and offering various services [2]. The increasing acknowledgment of DEA's value is reflected in numerous studies that have utilized this methodology to analyze hospital performance throughout the COVID-19 pandemic, yielding significant insights into the crisis's effects on efficiency and highlighting optimal practices for the allocation of resources [3].

The theoretical basis of DEA is grounded in the concept of efficiency frontiers, a principle first established in production theory to define the maximum possible output that can be achieved with a specific set of inputs. DEA modifies this idea for performance evaluation by employing linear programming techniques to empirically create these frontiers from observed data [7]. This methodology facilitates the calculation of productivity scores for each DMU, offering a comparative measure of efficiency against its counterparts. DEA models can be developed using either an input-focused or an output-focused approach [6]. Input-focused models aim to lower the inputs needed to reach a specified level of output, whereas output-focused models strive to enhance the outputs attainable with a given input set. The selection between these two methodologies is influenced by the particular context and the goals of the analysis.

A significant aspect of DEA modeling, especially within healthcare, is the integration of undesirable outputs into the evaluation. Conventional DEA models that concentrate solely on favorable outputs, such as patient satisfaction or positive treatment results, may yield deceptive findings by overlooking the negative externalities linked to healthcare delivery, including medical waste, patient readmissions, or hospital-acquired infections [8]. Therefore, a precise evaluation of hospital performance requires the integration of these undesirable outputs into the DEA framework. There are two main strategies for integrating undesirable outputs into DEA models: indirect approaches and direct approaches [7, 9].

Indirect methods generally involve converting undesirable outputs into desirable ones through uniform functions, such as taking the reciprocal or deducting the undesirable output from a constant. Conversely, direct methods integrate undesirable outputs directly into the production possibility set, enabling a more detailed representation of the trade-offs between favorable and unfavorable outcomes. Empirical studies have consistently shown that including undesirable outputs can significantly influence DMU performance rankings, underscoring the necessity of considering these factors in DEA-based assessments [10].

Expanding on this body of research, numerous studies have investigated different approaches for integrating undesirable outputs into DEA models. For example, the directional distance function facilitates the simultaneous enhancement of desirable outputs while lowering the number of undesirable ones, resulting in a holistic measure of efficiency that considers both positive and negative results [11, 12]. Additionally, Network DEA models have been created to deal with scenarios where undesirable outputs emerge as intermediate products in a multi-stage production process [12]. Moreover, researchers have examined how varying assumptions regarding the characteristics of undesirable outputs, such as whether they are fixed or variable-sum, impact the analysis [10, 13]. The concept of shadow pricing has also been used to evaluate both desirable and undesirable outputs, enabling a more thorough evaluation of efficiency that considers the relative worth of each outcome [3, 14].

The need for enhancing efficiency and economic performance within hospital systems has become increasingly urgent, driven by constantly rising healthcare costs and escalating demand for high-quality patient care [4,13]. Inefficient resource distribution and operational limitations can severely undermine a hospital's financial sustainability, thus impeding its ability to achieve optimal patient outcomes. DEA has emerged as a powerful and widely used methodology for benchmarking hospital performance and uncovering opportunities for improving resource utilization, ultimately fostering enhanced economic efficiency [15-17]. The ability of DEA to model complex relationships involving multiple inputs and outputs makes it especially appropriate for examining the intricate economic dynamics present in hospital environments. The financial implications of hospital efficiency have been further intensified by the unique challenges posed by the COVID-19 pandemic, which has placed significant financial strains on healthcare providers worldwide [3, 18, 19]. The pandemic has brought to light the critical need for resource optimization and the adoption of effective cost-containment strategies to secure the long-term economic viability of hospitals. DEA-focused studies in this context provide a crucial perspective into how hospitals can adjust their operational frameworks to sustain financial stability while offering essential services during times of acute crisis at the same time. Recent advancements in DEA methodologies are actively dealing with the inherent challenges posed by data uncertainty and the fluctuating nature of healthcare settings, thereby enabling more thorough and detailed evaluations of hospital economic performance. For example, fuzzy window DEA techniques facilitate the longitudinal examination of performance trends, accommodating the inherent variability and randomness typical of hospital operations [20]. Likewise, uncertain common-weights DEA models offer a structure for managing imprecise or incomplete data, resulting in more realistic assessments of efficiency in situations where data quality is limited [20, 21]. Additionally, the use of DEA extends to assessing economic efficiency in scientific and technological innovations within universities that contribute to the healthcare industry, thereby emphasizing the wider economic implications of research and development investments [22]. Beyond merely technical efficiency metrics, elements such as employee job satisfaction and the establishment of comprehensive performance appraisal systems significantly impact a hospital's overall economic performance [23].

An engaged and well-managed workforce is crucial for improving productivity, lowering operational costs, and cultivating a culture of continuous improvement. The combination of artificial neural networks with DEA methodologies can help pinpoint the primary factors affecting both efficiency and employee satisfaction, allowing hospitals to implement focused interventions aimed at optimizing economic results [24]. The use of advanced slack-based DEA models in related fields, such as evaluating Chinese commercial banks, further illustrates the flexibility and wide-ranging applicability of DEA in evaluating efficiency and economic performance across various sectors [25]. This research intends to assess hospital performance through DEA, taking into account both desirable and undesirable outputs. We employ a model that facilitates the evaluation of hospitals based on both input-oriented and output-oriented criteria. The significance of this research is highlighted by the potential for improved performance in emergency wards to lower mortality rates. DEA serves as a fundamental tool for efficiency analysis in multiple sectors, and recent developments continue to broaden its application scope [21]. A DEA-based methodology for evaluating hospital performance amid uncertainty has also been introduced. While DEA is a prevalent mathematical framework for estimating efficiency, its accuracy is contingent upon having complete data knowledge [10]. The anticipated findings are expected to offer actionable insights for hospital administrators and policymakers, guiding evidence-based decisions to improve the economic viability and long-term sustainability of healthcare systems. This research will advance the theoretical understanding of hospital efficiency and provide practical tools for improving economic results in a complex and evolving environment. Ultimately, this research aims to inform strategies for enhancing the efficiency and effectiveness of hospital emergency wards, with the overarching goal of decreasing mortality rates and improving patient outcomes. The structure of this paper is organized as follows: Section 2 offers a thorough literature review and theoretical background. Section 3 outlines the theoretical preliminaries and relevant literature on DEA and the management of undesirable outputs. Section 4 formulates the proposed DEA model and discusses the solution methodology. Section 5 presents a Case Study: Efficiency Analysis of Hospital Emergency wards in Tehran Province to demonstrate the model's application, and Section 6 wraps up the paper with a summary of key findings and suggestions for future research.

2. Literature review

Hospitals are pivotal to healthcare delivery, providing a comprehensive range of medical services, specialized treatments, and emergency care [3]. Their performance directly impacts the quality of healthcare services, a significance that was particularly evident during the COVID-19 pandemic, where hospital performance significantly affected patient mortality rates [23]. Consequently, the development of effective methods for evaluating hospital performance and productivity is a critical topic in healthcare literature.

Evaluating hospital performance is crucial for several reasons. To start with, it enables healthcare administrators and policymakers to pinpoint areas for enhancement and allocate resources effectively. Next, performance assessment facilitates benchmarking against other hospitals or industry standards, allowing for comparative analysis that can be the driving force behind improvements in healthcare delivery. By establishing robust assessment frameworks, healthcare professionals and policymakers can gain valuable perspectives into the strengths and shortcomings of individual hospitals, enabling informed decisions with respect to resource allocation, quality improvement initiatives, and patient care optimization [2, 23].

Traditional DEA models have been shown to be sensitive to undesirable factors and may produce non-unique optimal weights [26]. To address these limitations, researchers have developed robust DEA models that incorporate robust optimization techniques. The goal of these models is to make sure that performance assessments are reliable and sustainable, especially in undesirable environments [3]. This is essential for providing consistent and trustworthy results when evaluating hospital performance, where data can be subject to variability and inaccuracies.

The application of DEA extends beyond healthcare. For example, Lou et al. [22] analyzed the efficiency of Scientific and Technological Innovation (STI) in universities, determining an overall improvement in STI efficiency and pinpointing specific areas for further enhancement at the same time. This demonstrates the versatility of DEA as a performance evaluation tool. These diverse researches, as a whole, show that sensitivity and sustainability analyses are pivotal for obtaining a comprehensive insight into efficiency levels, pinpointing the key driving forces behind inefficiencies, and providing valuable perspectives for improving performance in different fields. The necessity for these analyses is vital, especially in complex systems like hospitals, where numerous factors can affect overall performance.

In summary, the literature highlights the vital role hospitals play in the healthcare system, the significance of robust performance assessment methods, and the shortcomings of traditional DEA models. The development of robust DEA models that can handle uncertainty is pivotal for providing reliable and actionable insights for improving hospital performance. This review underscores the need for further research in this area, especially in the context of the ongoing challenges facing healthcare systems globally.

3. Preliminaries

We consider n observations pertaining to n DMUs, which are entities that use inputs to produce outputs. $(x_j, y_j), j = 1,\ldots,n$. Let $x_j = (x_{1j},\ldots,x_{mj})^T$ & $y_j = (y_{1j},\ldots,y_{sj})$. All $x_j \in R^m$ & $y_j \in R^s$ & $x_j \succ 0,\ y_j \succ 0\ forj = 1,\ldots,n$. The input matrix *X* and output matrix *Y* can be represented as [10]:
$X = [x_1,\ldots,x_j,\ldots x_n]$ & $Y = [y_1,\ldots,y_j,\ldots y_n]$

Where *X* is an (m∗n) matrix and *Y* is an (s∗n) matrix.

The Production Possibility Set PPS, denoted by *T*, is generally defined as the set of all feasible input and output combinations: $T = \{(x,y)|x\ can\ produce\ y\}$ (1)

In DEA, under the assumption of Variable Returns to Scale, the PPS, denoted by *T*, is constructed using the observed input-output data. $(x_j, y_j)forj = 1,\ldots,n$. Following the formulation presented in [28]:

$$T = \left\{(x,y)\middle| x \geq \sum_{j=1}^{n}\lambda_j x_j\ ,\ y \leq \sum_{j=1}^{n}\lambda_j y_j\ ,\lambda_j \geq 0\ ,\ \sum_{j=1}^{n}\lambda_j = 1,\ j = 1,\ldots,n\right\}. \tag{2}$$

In the absence of undesirable input and output data, when a DMU_0, $o \in \{1,\ldots,n\}$ Is under evaluation, we can use the following BCC model 3:

$$\begin{aligned} &\theta_o^* = Min\theta_0 st.\\ &\sum_{j=1}^{n}\lambda_j x_{ij} \leq \theta_o x_{io} \qquad i = 1,\ldots,m\\ &\textstyle\sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{ro} \qquad r = 1,\ldots,s\\ &\textstyle\sum_{j=1}^{n}\lambda_j = 1 \qquad \lambda_j \geq 0 \qquad j = 1,\ldots,n \end{aligned} \tag{3}$$

Corresponding to each output *y, L(y)* is defined as follows:

$$L(y_j) = \{x|(x_j, y_j) \in T\} \tag{4}$$

In fact, $L(y_j)$ is a function in which y_j Pertains to a subset of inputs so that inputs can produce y_j.

Now, suppose that some inputs are undesirable. Therefore, input matrix *X* can be represented as $X = (X^D, X^U)^T$ where $X^D = (x_{1j}^D,\ldots,x_{m_1j}^D), j = 1,\ldots,n$ & $X^U = (x_{1j}^U,\ldots,x_{m_1j}^U)$ $j = 1,\ldots,n\ are\ (m_1 * n) and (m_2 * n)$ Matrixes that represent desirable and undesirable inputs, respectively. Similarly, suppose that some outputs are undesirable. In such a case, Matrix *Y* can be represented as $Y = (Y^D, Y^U)^T$, where $Y^D = (y_{1j}^D, \ldots, y_{s_1 j}^D), j = 1, \ldots, n$ & $Y^U = \left(y_{1j}^U, \ldots, y_{s_2 j}^U\right)$, $j = 1, \ldots, n$ are $(s_1 * n)$ and $(s_2 * n)$ Matrixes representing desirable (good) and undesirable (bad) inputs, respectively.

Definition 1: Let DMU_0 of $(x_1^D, x_1^U, y_1^D, y_1^U)$ Be dominant to DMU_0 of $(x_2^D, x_2^U, y_2^D, y_2^U)$ if $(x_1^D \leq x_2^D, x_1^U \geq x_2^U, y_1^D \geq y_2^D)$ & $y_2^U \geq y_1^U$ And the unequal be strict at least in a component. So that,

$$\begin{pmatrix} -x_1^D \\ x_1^U \\ y_1^D \\ -y_1^U \end{pmatrix} \geq \begin{pmatrix} -x_2^D \\ x_2^U \\ y_2^D \\ -y_2^U \end{pmatrix} \tag{5}$$

Definition 2: DMU_0 is efficient if in T there is no DMU to be dominant over it.

We consider the properties of the Production Possibility set (PPS) as follows:

(1) T is convex.

(2) T is closed.

(3) The monotony property of desirable inputs and outputs, so that:

$\forall u \in R_+^{m_1}, v \in R_+^{s_1}, (x^D, x^U, y^D, y^U) \in T \Rightarrow (x^D + u, x^U, y^D - v, y^U) \in T$

The inclusion of undesirable factors in the production process requires a careful consideration of PPS definition. Under the standard assumptions, the resulting PPS, T, may lack efficient DMUs, rendering the efficiency analysis problematic. To address this, we introduce a modified PPS, T, that satisfies properties (1) through (3), specifically designed to handle undesirable factors. The definition of this modified T is model 6:

$$T = \left\{ (x^D, x^U, y^D, y^U) \middle| \begin{matrix} x^D \geq \sum_{j=1}^n \lambda_j x_j^D, x^U = \sum_{j=1}^n \lambda_j x_j^U, y^U = \sum_{j=}^n \lambda_j y_j^U, y^D \leq \sum_{j=1}^n \lambda_j y_j^D \\ \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, \ j = 1, \ldots, n \end{matrix} \right\} \tag{6}$$

4. Methodology

Within the DEA framework, evaluating DMU efficiency is approached from either an input-oriented or an output-oriented perspective [27]. Input-oriented models, to enhance efficiency, seek to minimize inputs while maintaining or increasing outputs, reflecting a resource conservation strategy [28]. Conversely, output-oriented models are designed to maximize outputs while maintaining or decreasing inputs, emphasizing the optimization of production potential [29]. Specifically, in input-oriented models, the objective is to reduce the consumption of desirable inputs while simultaneously mitigating the generation of undesirable inputs (e.g., waste, pollution), reflecting a commitment to sustainable resource management. In contrast, output-oriented models strive to amplify the production of desirable outputs while minimizing the generation of undesirable outputs, aligning with the principles of environmentally conscious production [30].

Building upon the foundational work in DEA, Färe et al. [9] introduced a pioneering model designed to simultaneously increase desirable outputs and decrease undesirable outputs. However, this model suffered from a critical shortcoming: its inherent nonlinearity, which significantly complicates its practical application and computational tractability, particularly in large-scale datasets. Addressing this challenge, Ali and Seiford [8] proposed an alternative method that aims to simultaneously increase desirable outputs and decrease undesirable outputs. Nevertheless, this approach exhibits a dependence on value judgments, such that increased values can artificially inflate efficiency scores for inefficient DMUs, potentially leading to biased and misleading performance evaluations [25]. Recent study by Ghasemi et al., [31] has further highlighted the sensitivity of these value-dependent models to variations in data and the potential for spurious results.

Alternative approaches, such as the Window DEA (WD) method by Fare et al., [7] and the Multiplicative Leontief Transformation (MLT) method by Golany and Roll [14], have also been proposed to address the challenges of incorporating undesirable outputs in DEA models. However, a common shortcoming of these methods is that reductions in undesirable outputs are often coupled with unintended reductions in desirable outputs, potentially compromising overall performance and hindering the achievement of efficiency gains. This contradicts the fundamental principle of efficiency improvement, which should ideally involve an increase in desirable outputs or a decrease in undesirable outputs, or both, without sacrificing overall productivity [7,14]. Sueyoshi and Goto [32] have recently explored the trade-offs inherent in these methods and proposed alternative formulations that mitigate the risk of unintended reductions in desirable outputs.

Therefore, this research proposes novel DEA models that address these limitations by integrating both input-oriented and output-oriented approaches in the presence of undesirable factors. These models aim to provide a more comprehensive and robust evaluation of efficiency, accounting for the complex interrelationships between inputs, desirable outputs, and undesirable outputs. The proposed models will be evaluated using COVID-19 related data collected from the emergency wards of hospitals, allowing for a rigorous evaluation of their functionality, applicability, and potential to inform evidence-based decision-making in healthcare settings. This empirical evaluation will provide valuable insights into the performance of hospital emergency wards during a period of unprecedented stress and resource constraints. Detailed explanations of these proposed models will be provided in the subsequent sections, including mathematical formulations, algorithmic implementations, and sensitivity analyses to assess their robustness and reliability [27, 33]. Hosseinzadeh Lotfi et al., [34] have recently emphasized the importance of incorporating contextual factors and environmental variables in DEA models to account for the heterogeneity of hospital environments and improve the accuracy of efficiency assessments.

4.1 Nature of the input

Suppose $DMU_o = (x_o^D, x_o^U, y_o^D, y_o^U)$ to be the unit under evaluation corresponding to the output $y_o = (y_o^D, y_o^U)$, using (2) & $L(y_o^D, y_o^U)$ is defined as follows:

$$L(y_o^D, y_o^U) = \{(x^D, x^U) | (x^D, x^U, y_o^D, y_o^U) \in T\} \tag{7}$$

and we consider the subset of $L(y_o^D, y_o^U)$ as:

$$\partial^p L(y_o^D, y_o^U) = \{(x^D, x^U) | \forall (u, v) \geq 0, (u, v) \neq 0 \quad \Rightarrow \quad (x^D - u, x^U + v) \notin L(y_o^D, y_o^U)\} \tag{8}$$

where $\partial^s L(y_o^D, y_o^U)$ includes all inputs of the efficient DMUs capable of producing (y_o^D, y_o^U).

The model to evaluate the efficiency of DMU_0 with the maximum decrease in the value of x_o^D and the maximum increase in the value of x_o^U is as follows [6,27]:

$$d_o{}^D = x_o{}^D$$
$$d_o{}^U = x_o{}^U - x_{max}^U$$

So that:

$$(x_{max}^U)_i = max_j\{x_{ij}^U\}$$

Therefore, based on the definition of inefficiency, we have:

$$\theta_o^* = Max \quad \theta_o$$
$$st.$$
$$\sum_{j=1}^{n} \lambda_j x_j^D + s^- = x_o^D - \theta_o d_o{}^D$$

$$\sum_{j=1}^{n} \lambda_J x_j^U = x_o^U - \theta {d_o}^U$$
$$\sum_{j=1}^{n} \lambda_j y_j^D - s^+ = y_o^D$$
$$\sum_{j=1}^{n} \lambda_j y_j^U = y_o^U$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$\lambda_j \geq 0 \qquad for \qquad all \quad j = 1, \ldots, n \tag{9}$$

In accordance with the definition of production possibility set, model 1 is possible in this set.

Have the following:

Theorem 1: The DMU_0 in model 9 is efficient if and only if

1) $\theta_o^* = 1$

2) All slacks are zero for all optimal solutions.

Theorem 2: If all optimal solutions of model 9 are to be (θ^*, s^{-^*}), then $(x^D - \theta^* d^D - s^{-^*}, x^U - \theta^* d^U) \in \partial^p L(y_o^D, y_o^U)$

where s- is one of the optimal answers.

4.2 Nature of the output

Suppose $DMU_o = (x_o^D, x_o^U, y_o^D, y_o^U)$ is the unit under assessment corresponding to the input $x_o = (x_o^D, x_o^U)$. The outputs set $p(x_o^D, x_o^U)$ is defined as follows:

$p(x_o^D, x_o^U) = \{(y_o^D, y_o^U) | (x_o^D, x_o^U, y_o^D, y_o^U) \in T\}$

and the subset of $p(x_o^D, x_o^U)$ is taken to be:

$$\partial^p p(x_o^D, x_o^U) = \{(y_o^D, y_o^U) | \forall (u, v) \geq 0, (u, v) \neq 0 \;\; \Rightarrow \;\; (y_o^D + u, y_o^U - v) \notin p(x_o^D, x_o^U)\} \tag{10}$$

where $\partial^s L(y_o^D, y_o^U)$ includes all the inputs of the efficient DMUs that can produce (y_o^D, y_o^U).

The model used to evaluate the efficiency of DMU_0 with the most decrease of y_o^D and the most increase of y_o^U is as follows [18]:

$NE^d(x_0 + y_0) = sup\{\beta | y_0 + \beta d \in p(x_0)\}$

Where $d = (d^D, d^U)$ indicates the direction of the unit being evaluated such that $d^D \in R_+^{s_1}$ and $d^U \in R_-^{m_2}$ leads to increased desirable output and decreased undesirable output.

"This research employs a radial projection approach to optimize efficiency. Specifically, desirable outputs are radially projected onto the efficient frontier, while undesirable outputs are radially contracted. The scaling factors for these radial adjustments are defined as follows:

- Desirable outputs: $d^D = y_o^D$
- Undesirable outputs: $d^d = -y_o^U$

Therefore, according to the definition, we have:

$\beta_o^* = Max \quad \beta_o$

$st.$

$$\sum_{j=1}^{n} \lambda_j x_j^D + s^- = x_o^D$$

$$\sum_{j=1}^{n} \lambda_J x_j^U = x_o^U$$
$$\sum_{j=1}^{n} \lambda_j y_j^D - s^+ = y_o^D + \beta_o d_o^D$$
$$\sum_{j=1}^{n} \lambda_j y_j^U = y_o^U + \beta_o d_o^U$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$\lambda_j \geq 0 \quad for\ all \quad j = 1,\ldots,n \tag{11}$$

Theorem 3: The DMU_0 in model 11 is efficient if and only if:

1) $\beta_o^* = 1$
2) All slacks are zero for all optimal solutions.

Theorem 4: If β_o^* is the optimal solution of model 11 in DMU_0, then $(y_o^* + \beta_o^* d_o^D + s^{+^*}, y_o^U + \beta_o^* d_o^U) \in \partial^p p(x_o^D, x_o^U)$.

5. Case Study: Efficiency Analysis of Hospital Emergency wards in Tehran Province

This study investigates the efficiency of hospital emergency wards (EDs) in Tehran province of Iran, a context characterized by high patient volumes and resource constraints [3]. A substantial proportion of the population in Tehran province seeks medical care at public hospitals, leading to Emergency ward overcrowding, extended waiting times, and potential compromises in service quality [4]. These challenges can contribute to adverse patient outcomes, including patient dissatisfaction and, in severe cases, increased mortality, particularly among critically ill individuals [23]. To address these concerns, this research focuses on a sample of emergency centers from both public and private hospitals within Tehran province. Data was gathered prospectively over a one-month period, with queue times measured using dedicated queuing devices to guarantee accuracy and minimize observer bias.

The COVID-19 pandemic has further intensified these challenges, resulting in a significant surge in patient visits and mortality rates, thereby augmenting undesirable factors associated with patient mortality and resource consumption [3, 18, 24]. As of July 23, 2023, Iran had experienced over 7,466,311 confirmed cases and an estimated 146,837 deaths related to COVID-19. The pandemic has also precipitated shortages of essential medical equipment due to import restrictions and supply chain disruptions, necessitating the increased reliance on sterilization and reuse of equipment by hospitals [3, 5]. While this practice introduces potential risks, it is assumed that it does not fundamentally alter the production possibility set (PPS) or the feasibility status of the system under study. This assumption is based on the premise that the sterilization process, while resource-intensive, does not fundamentally change the relative efficiency with which inputs are transformed into outputs, given the constraints imposed by the pandemic and resource limitations [17, 35].

Several factors support this assertion

Fixed Technology: The core technology and processes for treating patients in the emergency ward remain largely unchanged. Sterilization, while an additional step, is primarily a supportive activity that ensures the continued safe operation of existing treatment protocols. It doesn't introduce fundamentally new treatment modalities or significantly alter the relationships between inputs (e.g., staff, beds, medications) and outputs (e.g., patients treated, discharged) [5, 36].

Capacity Constraints: The emergency wards are already operating at or near capacity due to the surge in number of patients. The primary constraint on output is not the availability of sterilized equipment (although that is a concern), but rather the overall capacity of the emergency ward to process patients given the available staff, beds, and other resources [4, 37]. Sterilization, therefore, acts as a necessary condition for maintaining existing capacity, rather than a driver of increased efficiency beyond that capacity [3, 38].

In this empirical research, a quantitative methodology was employed. The gathered data was analyzed using DEA, a non-parametric technique suitable for evaluating the relative efficiency of DMUs with multiple inputs and outputs [6]. Both input-oriented and output-oriented DEA models were applied to assess efficiency under different managerial perspectives: input-oriented models seeking to minimize inputs for a given level of output, and output-oriented models seeking to maximize outputs for a given level of input [27, 33].

For the numerical analysis, a sample of 30 DMUs, representing hospital emergency wards, was selected. The efficiency of these DMUs was evaluated based on five desirable inputs; X_1^D the number of nurses on shift, reflecting staffing levels; X_2^D the number of general practitioners; X_3^D the number of specialist or emergency medicine physicians, both representing physician resources; X_4^D the number of hospital beds, indicating capacity; and X_5^D patient waiting time in emergency wards, serving as a proxy for process efficiency. Furthermore, the inverse of adherence to medical equipment sterilization protocols prior to reuse was incorporated as an undesirable input X^U, reflecting potential iatrogenic risk [39].

These inputs were used to generate four desirable outputs; Y_1^D the number of patients discharged from the emergency department, representing successful treatment and throughput; Y_2^D the number of outpatients treated and discharged within 4 hours, reflecting rapid response capabilities; Y_3^D the number of patients with a length of stay between 4 and 12 hours; and Y_4^D the number of patients admitted and receiving services for more than 12 hours in the hospital, representing complex cases requiring extended care. The number of in-hospital deaths attributed to the presenting disease was considered an undesirable output Y^U, reflecting potential failures in treatment efficacy or patient management. Elevated values of Y^U indicate a failure to effectively manage patient conditions, negatively impacting overall performance.

Table 1

Emergency department data in Desirable and Undesirable input & output

DMU$_J$	Desirable Inputs					Undesirable input	Undesirable output	Desirable Outputs			
	X_1^D	X_2^D	X_3^D	X_4^D	X_5^D	X^U	Y^U	Y_1^D	Y_2^D	Y_3^D	Y_4^D
1	18	1	1	38	27	19	2	1155	295	265	32
2	19	2	1	41	15	4	3	1254	338	305	30
3	21	2	2	42	17	9	1	1259	325	261	28
4	19	2	1	39	21	14	4	1244	320	263	29
5	20	2	1	40	25	17	2	1254	323	271	29
6	22	2	2	42	34	7	7	917	125	169	22
7	21	2	1	41	26	15	3	1245	332	237	28
8	21	2	1	41	18	11	2	1254	323	270	28
9	20	2	1	40	19	8	1	1204	340	265	27
10	20	2	1	39	17	6	1	1254	315	270	29

Table 1
Continued

DMUJ	Desirable Inputs					Undesirable input	Undesirable output	Desirable Outputs			
	X_1^D	X_2^D	X_3^D	X_4^D	X_5^D	X^U	Y^U	Y_1^D	Y_2^D	Y_3^D	Y_4^D
11	20	2	1	39	18	10	2	1260	324	272	29
12	19	1	2	39	29	17	4	944	192	246	30
13	18	1	2	38	29	9	5	985	194	240	28
14	19	1	2	40	30	13	1	1085	295	226	32
15	19	1	2	39	30	12	7	764	162	116	20
16	20	1	2	41	31	5	5	691	150	244	19
17	20	1	2	42	31	8	3	994	192	246	28
18	20	1	2	41	25	9	4	931	201	256	29
19	21	1	2	42	26	18	2	941	188	274	28
20	20	1	2	41	25	16	1	1145	284	275	27
21	21	1	2	41	25	19	5	948	193	212	32
22	18	1	1	38	27	6	4	994	305	266	28
23	20	1	2	39	26	5	2	941	245	246	27
24	19	1	2	39	27	15	3	984	189	274	29
25	20	1	2	41	27	7	2	948	193	247	28
26	22	2	2	43	14	11	1	1259	335	271	30
27	23	2	2	44	32	5	6	1015	224	261	24
28	21	2	1	42	13	13	1	1370	365	322	35
29	22	2	1	42	15	18	2	1244	320	270	29
30	23	2	2	44	14	8	5	1154	314	272	31

Using GAMS software for both input-oriented and output-oriented models, the DEA analysis of the data was coded. The data presented in Table 1 was then reviewed and analyzed using this code to determine the efficiency of the 30 DMUs. A sample of the output from this program for DMUs 2, 3, and 29 is shown in Figures 1-3 as follows.

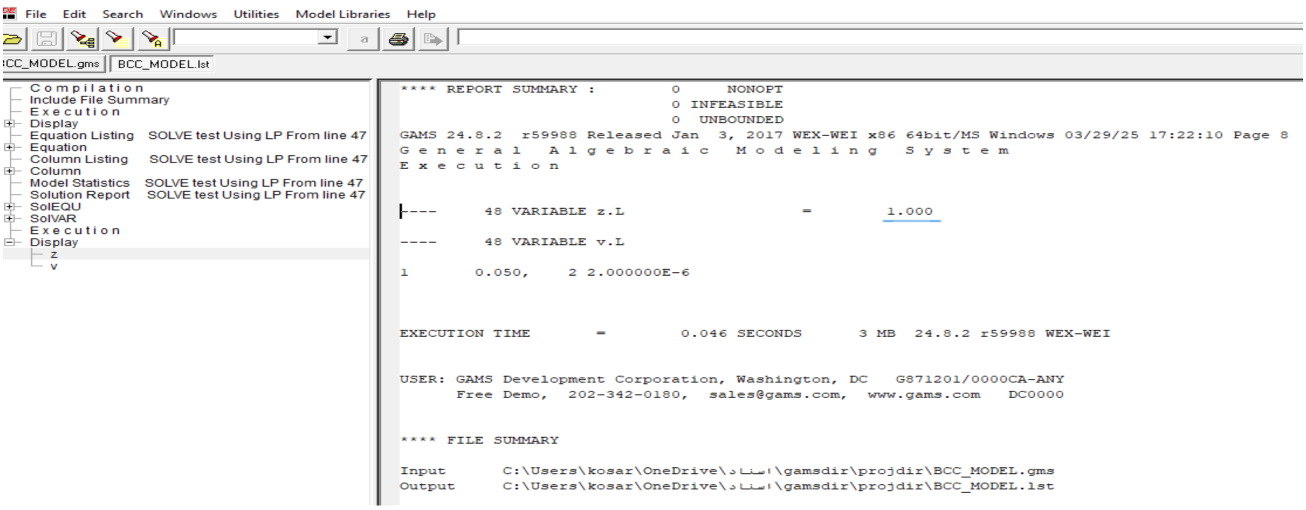

Fig. 1. Output of the GAMS software for DMU 2

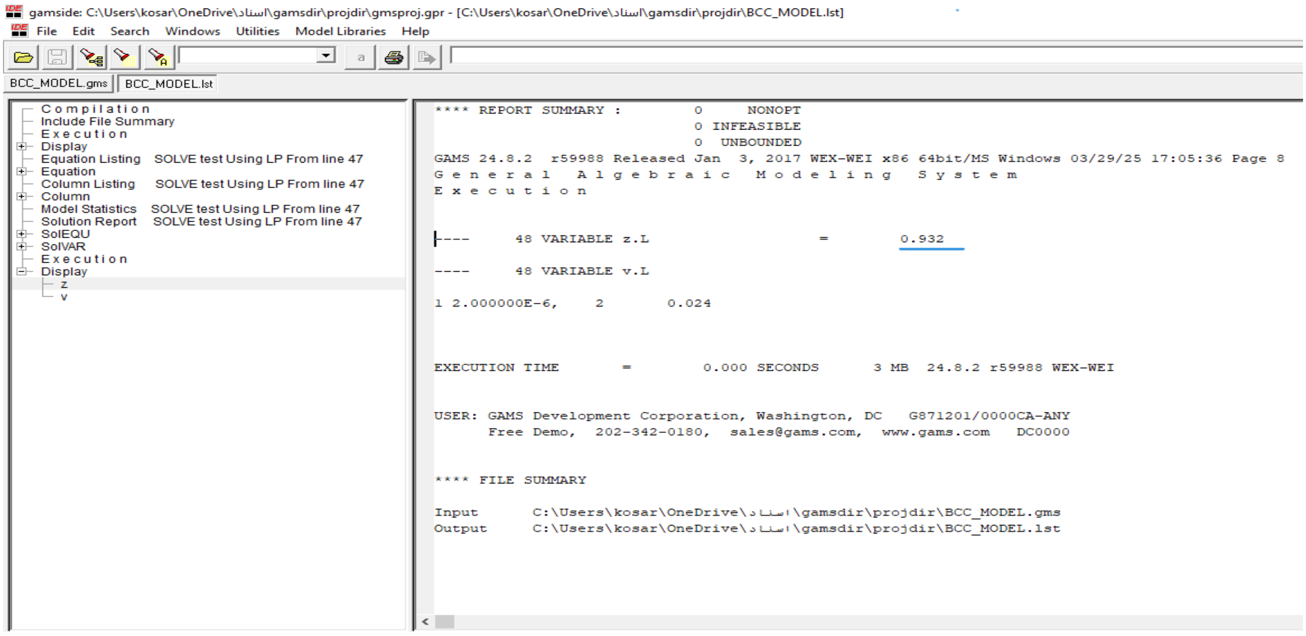

Fig. 2. Output of the GAMS software for DMU 3

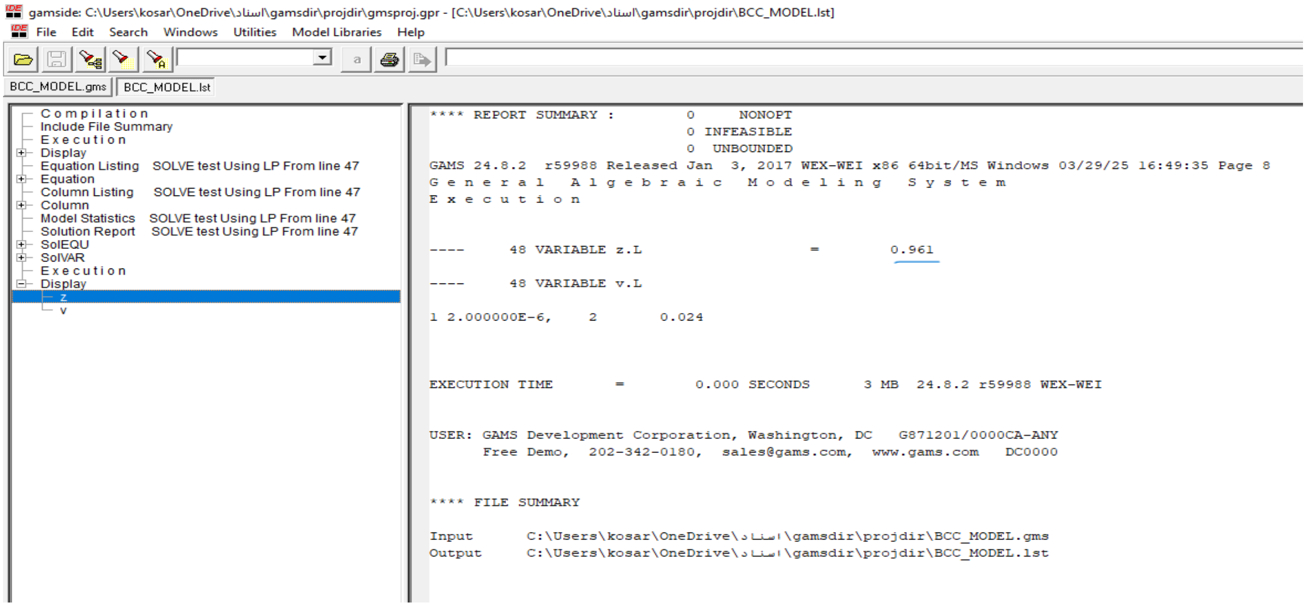

Fig. 3. Output of the GAMS software for DMU 29

The outputs of coding numbers 3, 9, and 19 are displayed in Tables 2 and 3 in a comparative and comprehensive manner.

Table 2
Impact of undesirable input on the proposed model's results

DMU_J	θ_o^*	BCC	DMU_J	θ_o^*	BCC	DMU_J	θ_o^*	BCC
1	1	1	11	0.9905	1	21	1	1
2	0.8952	1	12	1	0.9375	22	0.8859	1
3	0.9122	0.932	13	0.9163	0.9048	23	0.9663	1
4	0.9889	1	14	0.9963	1	24	0.9959	1
5	1	0.9711	15	0.7068	0.6615	25	0.8759	0.9169
6	0.7732	0.6693	16	0.7692	0.8873	26	0.9463	0.919
7	1	0.944	17	0.9039	0.9146	27	0.7901	0.8106
8	0.9477	0.9405	18	0.9898	1	28	0.9893	1
9	0.9763	1	19	1	0.9986	29	1	0.961
10	0.9689	1	20	1	1	30	0.7964	0.8857

Analysis of the results presented in Table 2 reveals a nuanced relationship between undesirable inputs and efficiency scores. Specifically, DMU_J exhibiting higher levels of undesirable inputs, such as 1, 20 & 21, demonstrate either maintained efficiency or, in cases such as 5 & 7, only a marginal decrease in efficiency. Conversely, some DMU_J that use fewer undesirable inputs than their counterparts have undergone a shift from efficient to inefficient status, or a reduction in their efficiency scores.

These findings suggest a counterintuitive impact: the presence of undesirable inputs within the evaluated DMUs appear related to an increase in efficiency, under the caveat that the sterilization of equipment is limited to two or three times to maintain the feasibility of the system. This implies that excessive use of the undesirable input to circumvent the PPS is not a viable strategy. Conversely, a reduction in undesirable inputs does not necessarily lead to an increase in efficiency. This observation highlights the complex interplay between desirable and undesirable factors in determining the overall efficiency.

Therefore, we conclude that the presence of undesirable inputs, within the defined operational constraints, has a significant influence on the efficiency scores of hospital emergency wards. This finding is consistent with the real-world challenges faced by hospitals, where resource constraints and the need to reuse equipment can influence operational efficiency. This empirical evidence supports the validity and functionality of the proposed model, as well as the underlying theoretical framework.

Table 3
Undesirable results generated by the proposed model

DMU_J	β_o^*	BCC	DMU_J	β_o^*	BCC	DMU_J	β_o^*	BCC
1	1	1	11	0.9905	1	21	0.9663	1
2	0.9994	1	12	0.8457	0.9375	22	0.9859	1
3	1	0.932	13	0.7548	0.9048	23	1	1
4	0.8997	1	14	1	1	24	0.9959	1
5	0.969	0.9711	15	0.5568	0.6615	25	0.9759	0.9169
6	0.5782	0.6693	16	0.7895	0.8873	26	1	0.919
7	0.9108	0.944	17	0.9036	0.9146	27	0.7801	0.8106
8	0.9377	0.9405	18	0.9818	1	28	1	1
9	1	1	19	1	0.9986	29	0.9658	0.961
10	1	1	20	1	1	30	0.7324	0.8857

Analysis of the results presented in Table 3 shows a clear inverse relationship between undesirable outputs and efficiency scores. Specifically, DMU_J exhibiting higher levels of undesirable outputs, such as DMU_2 and DMU_4, demonstrate decreased efficiency, with some transitioning from efficient to inefficient status. Conversely, the opposite trend is also evident. Notably, several DMUs, including 3, 19, 23 & 26, initially classified as inefficient under the output-oriented BCC model with variable returns to scale, have shifted from inefficient to efficient status due to lower mortality rates in these hospitals.

"These findings robustly support the conclusion that higher levels of undesirable outputs within the assessed DMU_J are significantly and negatively correlated with efficiency scores, and conversely, lower levels of undesirable outputs are associated with improved efficiency. This observation underscores the substantial impact of undesirable outputs on the overall efficiency assessment, highlighting the critical importance of their inclusion in performance evaluations. The results are consistent with the real-world dynamics of hospital emergency wards, where mortality rates serve as a key performance indicator reflecting the quality of care and the effectiveness of resource utilization. This empirical evidence further supports the validity and applicability of the proposed model, confirming its accurate reflection of these complex real-world relationships.

Furthermore, the implications of increased mortality rates extend far beyond purely technical efficiency metrics, impacting critical aspects of organizational and societal well-being. Elevated mortality rates within hospital emergency wards can lead to a significant erosion of social capital, as public trust in the healthcare system diminishes and individuals become less willing to seek timely medical care. This decline in social capital can have far-reaching consequences, affecting community cohesion, civic engagement, and overall societal resilience.

Moreover, increased mortality rates can negatively impact the availability and quality of human resources within the healthcare sector. The loss of experienced medical professionals due to burnout, stress, or disillusionment can intensify the existing staffing shortages and compromise the ability of hospitals to deliver high-quality care. This attrition of human capital can further contribute to a decline in the morale of remaining healthcare providers, leading to reduced job satisfaction, increased absenteeism, and decreased productivity. The cumulative effect of these factors can create a vicious cycle, where declining morale and workforce shortages further contribute to increased mortality rates and decreased efficiency.

Ultimately, the depletion of social capital and human resources resulting from increased mortality rates can have a significant and detrimental impact on the economic productivity of both the organization and the broader community. Reduced productivity, increased absenteeism, and the costs associated with recruiting and training new staff can all contribute to a decline in economic output. Furthermore, the loss of productive members of society due to preventable deaths represents a significant economic cost, reducing the overall potential for economic growth and development. Therefore, the findings of this research underscore the critical importance of addressing the root causes of undesirable outputs in hospital emergency wards, not only to improve technical efficiency but also to safeguard social capital, protect human resources, and promote long-term economic prosperity.

Limitations

1. Limited Data Availability Due to Lack of Managerial Cooperation: "A significant limitation of this study was the difficulty in obtaining complete datasets from all participating hospital emergency wards. This was primarily due to inconsistent managerial cooperation in providing timely and comprehensive data, potentially introducing bias or limiting the generalizability of the findings."

2. Time-Intensive Data Collection Process: "The extensive time required for data collection posed another constraint. The manual extraction and compilation of data from various emergency department records proved to be a labor-intensive and time-consuming process, ultimately restricting the scope of the study and the timeframe for analysis."

6. Conclusions

Amidst the global COVID-19 pandemic, characterized by widespread infection and strained healthcare systems, this study aimed to address critical objectives: reducing patient waiting times in hospital emergency rooms, identifying key queuing points and essential resources, optimizing resource allocation, and ultimately, reducing patient mortality.

This study addressed a salient gap in the extant literature by developing a refined methodological framework for evaluating the efficiency of DMUs operating within complex environments characterized by the simultaneous presence of desirable and undesirable inputs and outputs. The central objective was to formulate, rigorously validate, and disseminate an enhanced DEA model specifically tailored to accommodate such multifaceted scenarios. The resultant model was subsequently deployed in a case study examining the efficiency of emergency wards across a cohort of 30 hospitals in Tehran province, thereby providing empirical validation of its applicability within a real-world healthcare context.

The findings derived from the case study unequivocally affirm the validity and practical utility of the proposed DEA model within organizational and hospital settings. By identifying and benchmarking exemplar DMUs exhibiting superior efficiency, the model furnishes hospital management with actionable intelligence to facilitate performance enhancement across both efficient and inefficient units within their emergency wards. This, in turn, puts at risk the improvements in the quality of emergency care provision, optimization of resource allocation, and augmentation of overall economic productivity within the hospital system.

A pivotal contribution of the developed model resides in its explicit and nuanced consideration of both desirable and undesirable inputs and outputs. This capability empowers organizations to formulate targeted strategies for the mitigation of undesirable elements, thereby enhancing overall operational efficiency and minimizing potential economic losses. The model distinguishes itself from prior DEA methodologies through its capacity to effectively address situations wherein DMUs generate undesirable outputs, affording a more realistic and comprehensive assessment of performance. The explicit emphasis on simultaneously addressing both desirable and undesirable inputs and outputs constitutes a significant advancement in the field of efficiency analysis. For instance, the model's applicability extends to the recycling industry, wherein the effective management of both undesirable inputs (e.g., contaminated recyclable materials) and undesirable outputs (e.g., residual waste streams) is of paramount importance.

6.1 Method validation and its applications

The proposed modified DEA model exhibits broad applicability across a diverse spectrum of industries characterized by the intricate management of both desirable and undesirable inputs and outputs. Within the manufacturing sector, for example, production units can be evaluated utilizing raw materials, energy consumption, labor inputs, and capital investments as desirable inputs; finished goods production and revenue generation as desirable outputs; and waste generation and carbon emissions as undesirable outputs. The enhanced model developed in this research paper can generate actionable scenarios for augmenting efficiency through the simultaneous maximization of desirable outputs and minimization of undesirable outputs. Analogously, within the realm of environmental management, the model can be utilized to evaluate the performance of entities such as water treatment facilities or recycling plants, focusing on the maximization of environmental benefits (e.g., potable water production, recycled material recovery) while concurrently minimizing waste generation and pollutant discharge. Further potential applications encompass energy generation (where power plants strive to maximize electricity production while minimizing greenhouse gas emissions), agricultural production (optimizing crop yields while minimizing fertilizer runoff), construction activities (maximizing building output while reducing material waste and environmental impact), tourism operations (enhancing visitor experiences while minimizing ecological degradation), and a wide array of sectors including extractive industries, mining operations, and waste management systems. This inherent versatility underscores the model's potential as a valuable analytical tool for evaluating and enhancing efficiency across many industries and organizational contexts, ultimately contributing to improved resource allocation, enhanced economic productivity, and sustainable operational practices.

Author Contributions

Conceptualization, A.M. and B.D.; methodology, A.M.; software, G.T.; validation, A.M. and B.D.; formal analysis, A.M.; investigation, A.M.; resources, G.T. and S.R.; data curation, A.M.; writing—original draft preparation, A.M.; writing—review and editing, B.D. and M.S.; visualization, S.R.; supervision, B.D. and G.T.; project administration, B.D. , G.T. and M.S.; All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

All data generated or analyzed during this study are included in this published article.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

All authors of the manuscript honestly declare that we have no competing interests related to this work and during the research. Our research is independent and not influenced by external factors such as financial resources, personal or professional relationships, or political affiliations. In carrying out this work and the stages of the research, we have acted with honesty and avoiding bias and by scientific techniques and ethics. By researching and analyzing the results, we have strictly adhered to the principles of academic integrity and ethical behavior throughout this project.

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