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Revolutionizing Decision-Making in E-Commerce and IT Procurement: An IVPNS-COBRA Linguistic Variable Framework for Enhanced Multi-Criteria Analysis

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ABSTRACT

Real-world challenges in e-commerce strategy selection and IT supplier evaluation are inherently complex due to uncertainty and incomplete information, necessitating a multi-faceted approach for effective decision-making. Traditional single methods often fail to address these complexities adequately. To overcome this limitation, this research introduces an advanced methodology by integrating the Interval-Valued Pythagorean Neutrosophic Set (IVPNS) with the Comprehensive Distance-Based Ranking (COBRA) approach, enhancing the handling of indeterminate information related to truth, falsity, and uncertainty. IVPNS provides a robust mathematical framework for managing ambiguity, a common challenge in Multi-Criteria Decision-Making (MCDM) across various domains. Prior to this study, IVPNS lacked a linguistic variable—a crucial component for expressing human judgments. To bridge this gap, we enhance the IVPNS-COBRA framework by incorporating 5-point and 7-point linguistic scales, ensuring compliance with established IVPNS conditions. Additionally, we introduce two distance measures, Euclidean and Hamming distances, to refine alternative evaluations. The proposed IVPNS-COBRA method is validated through real-world applications, including the evaluation of three e-commerce development strategies (assessed against five criteria) and four IT supplier selection alternatives (evaluated using nine criteria). The results demonstrate the reliability of this MCDM model, providing decision-makers with a more precise and structured approach for selecting optimal e-commerce strategies and IT suppliers in complex environments.

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1. Introduction

The rapid growth of e-commerce has fundamentally transformed business operations, influencing trade efficiency, customer engagement, and competitive strategies. As digitalization becomes central, companies increasingly rely on innovative e-commerce strategies to optimize their market presence and operational effectiveness. These strategies include technological integration, consumer behavior analysis, regulatory adaptation, and competitive positioning. Similarly, in the digital economy, IT supplier selection plays a crucial role in ensuring operational efficiency, cybersecurity, and business continuity. As companies depend more on digital tools and services, selecting the right IT supplier becomes a strategic necessity, requiring a comprehensive evaluation of cost, quality, risk, and innovation. Given the complexities involved in selecting optimal e-commerce strategies and IT suppliers, Multi-Criteria Decision-Making (MCDM) methodologies have gained prominence in addressing these challenges.

1.1 E-Commerce and IT Supplier Selection

E-commerce has fundamentally transformed global trade by expanding market access, lowering transaction costs, and enhancing customer experiences. This rapid expansion has significantly reshaped the economic landscape by altering traditional business structures, increasing market reach, and improving trade efficiency. Scholars have examined various e-commerce growth strategies, emphasizing technological adoption, consumer behavior, regulatory policies, and market competition as key factors influencing its impact.

A crucial driver of e-commerce success is the integration of advanced technologies. According to [1], digital innovations such as blockchain, cloud computing, and artificial intelligence enhance transaction security and operational effectiveness. Similarly, [2] highlights how automation and data analytics refine decision-making, allowing businesses to personalize offerings based on consumer preferences. Alongside technological advancements, understanding consumer behavior is vital for developing effective e-commerce strategies. Briandana *et al*., [3] assert that competitive pricing, personalized marketing, and the convenience of online shopping significantly influence purchasing decisions. Furthermore, Boadu, [4] emphasize the role of customer relationship management (CRM) systems in fostering customer loyalty and retention.

E-commerce has also intensified market competition by lowering entry barriers and driving innovation. According to [5] they, they explain that online competition compels firms to adopt strategic approaches such as cost leadership, niche targeting, and differentiation. The economic impact of e-commerce varies depending on the business model. In 2006 [6], he categorizes models like Business-to-Business (B2B), Business-to-Consumer (B2C), and Consumer-to-Consumer (C2C) critical determinants of e-commerce's economic influence.

In parallel, selecting the right IT supplier has become a strategic business decision, directly affecting competitiveness, cost management, and operational efficiency. Companies increasingly rely on multi-criteria decision-making (MCDM) techniques and economic theories to assess suppliers based on factors such as cost, quality, risk, and innovation. Transaction Cost Economics (TCE) [7] suggests that firms prioritize suppliers who minimize costs and maximize value. Additionally, with a growing emphasis on sustainability, businesses now consider Environmental, Social, and Governance (ESG) criteria in supplier selection [8]. The triple-bottom-line framework, which evaluates social, environmental, and economic aspects, plays a crucial role in this decision-making process.

Supplier selection in the IT sector requires balancing qualitative and quantitative factors. Researchers have introduced various fuzzy MCDM methods to address subjectivity and uncertainty in supplier evaluation. Razak *et al.,* [9] proposes a Fuzzy TOPSIS approach with sub-criteria assessments, while Sobhanallahi *et al*., [10] suggests a hybrid QFD-TOPSIS model. Taghipour *et al*., [11] integrate intuitionistic Fuzzy VIKOR and Q-ROF TOPSIS techniques to enhance selection processes by prioritizing key factors such as quality, cost, performance, and flexibility. Implementing these methodologies enables organizations to mitigate supplier-related risks and strengthen their competitive advantage in the global market [12,10].

1.2 Pythagorean Sets and Neutrosophic Sets

Neutrosophic set theory and Pythagorean fuzzy set theory represent significant advancements beyond traditional fuzzy sets and have recently attracted considerable attention. The Neutrosophic set theory founded by [13] expands upon fuzzy sets and intuitionistic fuzzy sets that can deal with uncertainty, imprecise, incomplete, and inconsistent information that cannot adequately handle by traditional methods. Neutrosophic set theory, emphasizing its three key aspects: truth-membership, indeterminacy-membership, and falsity-membership. On the other hand, Pythagorean fuzzy set theory, proposed by Zhang *et al.,* [14], extends traditional fuzzy sets by offering a more flexible approach to modeling uncertainty and inconsistency by Yang *et al*., [15]. Neutrosophic set theory also extends classical set concepts, including fuzzy sets [16], interval-valued fuzzy sets [17], intuitionistic fuzzy sets [18], and interval intuitionistic fuzzy sets [19].

In the context of neutrosophic set theory, [20] introduced the concept of simplified neutrosophic sets (SNS) as a subclass of neutrosophic sets, providing a framework for handling simplified indeterminate and inconsistent information. Furthermore, *Iryna et al*., [21] proposed a single-valued neutrosophic set (SVNS) method with a weighted correlation coefficient for fault diagnosis, demonstrating the practical applications of neutrosophic sets in real-world problem-solving. Smarandache *et al*., [22] introduce new concepts in this area, such as neutrosophic-b-open sets and neutrosophic g*-closed sets, respectively. These concepts are further explored in the context of neutrosophic bitopological and neutrosophic topological spaces.

Neutrosophic set theory, which focuses on handling imprecision and uncertainty, has been the subject of several recent studies. Various applications of neutrosophic sets in mathematical contexts have been explored. For example, [23] introduced neutrosophic b-open sets and their properties in neutrosophic topological spaces. [24] extended this work by introducing the concept of neutrosophic soft expert sets and their basic operations, focusing on multiple criteria decision-making applications. [25] further generalized the concept to neutrosophic 2-metric spaces, proving various fixed-point theorems. Al-Tahan *et al*., [26] combined the notions of ordered algebraic structures and neutrosophy, defining single-valued neutrosophic sets in ordered groupoids and exploring their properties. These studies collectively demonstrate the versatility and potential of neutrosophic sets in addressing uncertainty and inconsistency in various mathematical and decision-making contexts. The concept of interval valued neutrosophic sets has been applied to various fields, including graph theory [27], topological spaces [28], medical diagnosis [29], and subgroup theory [30]. These applications have expanded the understanding and use of interval valued neutrosophic sets, demonstrating their potential in a wide range of disciplines.

Pythagorean fuzzy set theory has experienced considerable development and application across various domains. Zhang *et al*., [31] introduced new similarity measures for Pythagorean fuzzy sets to tackle the challenge of distinguishing highly similar but inconsistent sets, thereby enhancing the accuracy of similarity calculations in practical applications. According to [32], they investigated the distance measurement of Interval-valued Pythagorean Fuzzy Sets (IVPFSs) and their related applications, addressing unresolved issues in the field. Numerous studies have delved into the concept of Interval Valued Pythagorean Sets (IVPS). For example, [33] presented a correlation measure for IVPS, while [34] defined interval-valued Pythagorean fuzzy ideals in semigroups. Du *et al*., [35] proposed a method for multi-attribute decision making using interval-valued Pythagorean fuzzy linguistic information. Collectively, these studies enhance the understanding and application of IVPS in various fields

Both neutrosophic set theory and Pythagorean fuzzy set theory have found applications across different domains. For instance,[36] proposed a fault diagnosis method based on attributes weighted neutrosophic sets, demonstrating the utility of neutrosophic sets in coping with uncertain information. Similarly, Yang *et al*., [15] developed hesitant Pythagorean fuzzy interaction aggregation operators for multiple attribute decision-making, showcasing the practical relevance of Pythagorean fuzzy set theory. Haq *et al*., [37] present a novel framework that combines Interval-Valued Neutrosophic Sets (IVNSs) with the entropy-MultiAtributive Ideal-Real Comparative Analysis (MAIRCA) method, while Ismail *et al*., [38] introduce an integrated Pythagorean Neutrosophic Set (PNS) and Decision-Making Trial and Evaluation Laboratory (DEMATEL) approach to tackle uncertainty and linguistic vagueness in multi-criteria decision-making, improving the accuracy and validity of expert judgments through an eight-step process that includes a new linguistic variable formulation and its use in analyzing causal relationships in halal certification barriers. Kamari *et al*., [39] proposed the integration of the Pythagorean Neutrosophic Set (PNS) with the Method Based on the Removal Effects of Criteria (MEREC). Additionally, they introduced 5-point, 9-point, and 11-point PNS linguistic variables, which can be used to effectively capture and represent expert evaluations. Biswas *et al*., [40] present a neutrosophic fuzzy decision-making framework for selecting the best canteen location on a university campus. Their study utilizes a multi-criteria decision-making (MCDM) approach, where the CRITIC method is used to assign weights to evaluation criteria, and the COPRAS method ranks the available alternatives. Similarly, Basuri *et al*., [40] examine the sustainable location problem for school site selection by employing a decision-analytics approach that integrates neutrosophic numbers with MCDM techniques. This study also applies the CRITIC method to determine the importance of various criteria and sub-criteria, while the COPRAS method ranks the potential school locations.The integration of IVPNS with the COBRA ranking method aligns with recent advancements in decision-analytics-based sustainable location problems, where MCDM techniques such as CRITIC-COPRAS have been effectively employed to enhance selection processes under uncertainty [41]. Furthermore, the adoption of neutrosophic-based decision models has been increasingly recognized in healthcare and other critical domains, as demonstrated by the utilization of Interval-Valued Fermatean Neutrosophic Super HyperSoft Sets for complex decision-making scenarios [42]. This approach is designed to handle objective criteria with precise inputs and subjective criteria with ambiguous or uncertain information simultaneously.

1.3 Comprehensive Distance-Based Ranking

The Comprehensive Distance Based Ranking (COBRA) method is an advanced decision-making tool used in Multiple Criteria Decision Making (MCDM) processes. COBRA is designed to evaluate and rank alternatives by calculating the distances between the options and a predefined ideal solution. This method ensures that the most optimal alternative is selected based on the smallest distance to the ideal solution, thereby providing a robust framework for decision-making. Krstić *et al*., [43] applied the COBRA method to assess the applicability of Industry 4.0 technologies in reverse logistics, demonstrating its effectiveness in ranking technologies within a circular economy framework. This application showcased the potential of the COBRA method in addressing complex decision-making scenarios in supply chain and sustainability domains.

The study employed the COBRA method for the final assessment and ranking of alternatives, highlighting its relevance in facilitating strategic decision-making processes in e-commerce development. In recent literature, COBRA has been integrated with other mathematical and decision-making frameworks to enhance its applicability and accuracy. For instance, Krstić *et al*., [43] combined the Best-Worst Method (BWM) with the COBRA method to rank technologies, illustrating the adaptability of the COBRA method in conjunction with established MCDM models.

Popovic *et al*., [44] integrates the COBRA method by combining method based on the Removal Effect of Criteria (MEREC) and COBRA, offering a sophisticated tool for addressing the inherent uncertainties and complexities in e-commerce strategy development. In their paper, they used MEREC to find the weight for each criterion and the COBRA method for ranking alternatives. By validating the model through practical application, the study underscores its potential to enhance decision-making processes in various domains beyond e-commerce. These integrations have broadened COBRA's applicability, making it a versatile tool for addressing diverse decision-making challenges.

As e-commerce development and IT supplier selection involve complex and uncertain decision-making processes, recent advancements in Neutrosophic set theory have introduced the concept of Interval-Valued Pythagorean Neutrosophic Sets (IVPNS). While Pythagorean Neutrosophic Sets (PNS) [45] have already extended conventional Neutrosophic sets, the integration of IVPNS with the COBRA method remains an unexplored domain. This study addresses this gap by proposing a fusion of IVPNS with COBRA to enhance the efficacy of MCDM in e-commerce strategy selection and IT supplier evaluation. The integration of IVPNS into COBRA is designed to achieve a more robust ranking of alternatives, improving decision-making outcomes for businesses navigating the complexities of e-commerce strategies and IT supplier selection. A novel linguistic variable under the IVPNS environment has been developed, enabling a greater range of values for membership functions. This linguistic variable considers five- and seven-scale assessments, allowing decision-makers to handle the uncertainty inherent in selecting the best e-commerce strategies and IT suppliers.

Moreover, COBRA’s recent incorporation with IVPNS has expanded its capabilities by addressing the challenge of indeterminate and inconsistent information, which is a common issue in MCDM problems. By combining IVPNS’s ability to handle uncertain data with COBRA’s robust ranking mechanism, businesses can achieve more precise and comprehensive evaluations of their strategic choices. This integration is particularly valuable in e-commerce development, where companies must weigh factors such as market trends, consumer behavior, and technological advancements to optimize their strategies. Similarly, in IT supplier selection, this approach enables organizations to assess suppliers more accurately based on criteria such as cost efficiency, innovation, and risk mitigation. Studies have validated this integrated approach, demonstrating its effectiveness in handling complex, uncertain data in both e-commerce and IT supplier evaluation scenarios. By leveraging IVPNS within the COBRA framework, decision-makers can refine their selection processes, ensuring that chosen strategies and suppliers align with their business objectives. This synergy between COBRA and IVPNS represents significant advancement in the field of decision-making, offering a powerful tool for evaluating and ranking alternatives across various domains. As businesses continue to navigate the challenges of digital transformation, the integration of IVPNS with COBRA provides a systematic and structured approach to making strategic e-commerce and IT supplier decisions, ensuring long-term sustainability and competitive advantage.

The remainder of the paper is organized as follows: The following section briefly some fundamental concepts of IVPFS, and INS. Section 3 covers the proposed method, detailing the linguistic variable for IVPNS and demonstrating its validity in meeting the established conditions for

IVPNS. In section 4, it introduces and develops the proposed IVPNS-COBRA method. Section 5 discusses the implementation of the proposed method in terms of the 5 and 7-Scale of linguistic variables, finally, Section 6, describes the findings and proposal for future research.

2. Preliminaries

This section introduces the basic definitions of the Interval Valued Pythagorean Neutrosophic set (IVPNS).

2.1 Interval Valued Pythagorean Fuzzy Number

Definition 1. Let x be an ordinary finite nonempty set, then a IVPFS $\tilde{p}$ in X is a defined as:

$$\tilde{P} = \{< x, \tilde{P}(\tilde{\mu}_{\tilde{P}}(x), \tilde{v}_{\tilde{P}}(x) > x \in X\}$$

Where the function $\tilde{\mu}_{\tilde{P}}(x) \subseteq [0,1]$ and $\tilde{v}_{\tilde{P}}(x) \subseteq [0,1]$ are interval values, the lower and upper interval $\tilde{\mu}_{\tilde{P}}(x)$ are $\tilde{\mu}^L_{\tilde{P}}(x)$ and $\tilde{\mu}^U_{\tilde{P}}(x)$, while the lower and upper interval $\tilde{v}_{\tilde{P}}(x)$ are $\tilde{v}^L_{\tilde{P}}(x)$ and $\tilde{v}^U_{\tilde{P}}(x)$, respectively and satisfy :

$$\tilde{\mu}^U_{\tilde{P}}(x) + \tilde{v}^U_{\tilde{P}}(x) \leq 1 \tag{1}$$

and

$$(\tilde{\mu}^U_{\tilde{P}}(x))^2 + (\tilde{v}^U_{\tilde{P}}(x))^2 \leq 1 \tag{2}$$

For every $x \in X$, $\tilde{\pi}_{\tilde{P}}(x) = [\tilde{\pi}^L_{\tilde{P}}(x), \tilde{\pi}^U_{\tilde{P}}(x)]$ is called a Pythagorean index of x to $\tilde{P}$, where $\tilde{\pi}^U_{\tilde{P}} = \sqrt{1-(\tilde{\pi}^U_{\tilde{P}}(x))^2 - (\tilde{\pi}^U_{\tilde{P}}(x))^2}, \tilde{\pi}^L_{\tilde{P}} = \sqrt{1-(\tilde{\pi}^L_{\tilde{P}}(x))^2 - (\tilde{\pi}^L_{\tilde{P}}(x))^2}$.

2.2 Interval Valued Valued Neutrosophic Set

Definition 2. Let U be a universe of discourse (objects) with generic elements in U denote by x. Then the interval Neutrosophic set (IVNS) A in x is characterized by the truth-membership function $b_A(x)$, indeterminacy membership function $I_A(x)$ and falsity-membership function $s_A(x)$. Every point x in X , we have that

$b_A(x) = [\inf b_A(x), \sup b_A(x)],\ \mathrm{I}_A(x) = [\inf I_A(x), \sup I_A(x)]$, and $s_A(x) = [\inf s_A(x), \sup s_A(x)] \subset [0,1]$

and

$$0 \leq \sup b_A(x) + \sup I_A(x) \sup s_A(x)] \leq 3, x \in U \tag{3}$$

and every function lies between [0,1] in U .

For convenience, can use $x = [b^L_A, b^U_A], [I^L_A, I^U_A], [s^L_A, s^U_A]$ to represent an element of INS.

2.3 Interval Value Pythagorean Neutrosophic Set (IVPNS)

In this section, we present Interval value Pyhtagorean Neutrosophic set (IVPNS), its algebraic operations and their properties according to Razak *et al*. [46].

Definition 3. Let U be a universe of discourse (objects) with generic elements in U denote by x. Then the IVPNS of $\tilde{A}$ is an object having the for

$$\tilde{A} = \left\{ < x, [(\tilde{b}^L_{\tilde{A}}(x), \tilde{b}^U_{\tilde{A}}(x)], [I^L_{\tilde{A}}(x), I^U_{\tilde{A}}(x)], [\tilde{b}^L_{\tilde{A}}(x), \tilde{s}^U_{\tilde{A}}(x)] > \middle| x \in X \right\} \tag{4}$$

Then the IVPNS $\tilde{A}$ in x is characteristics by truth-membership function $\tilde{b}_{\tilde{A}}(x)$, indeterminacy membership function $\tilde{I}_{\tilde{A}}(x)$ and falsity-membership function $\tilde{s}_{\tilde{A}}(x)$. For each point x in X , the lower and upper intervals are $\tilde{b}_{\tilde{A}}(x) = [\tilde{b}^L_{\tilde{A}}(x), \tilde{b}^U_{\tilde{A}}(x)] \subseteq [0,1]$, $I_{\tilde{A}}(x) = [I^L_{\tilde{A}}(x), I^U_{\tilde{A}}(x)] \subseteq [0,1]$, and

$\tilde{s}_{\tilde{A}}(x)=[\tilde{b}_{\tilde{A}}^{L}(x),\tilde{s}_{\tilde{A}}^{U}(x)]\subseteq[0,1]$, respectively. The $\tilde{b}_{\tilde{A}}(x)$ and $\tilde{I}_{\tilde{A}}(x)$ are dependent on neutrosophic components and $\tilde{s}_{\tilde{A}}(x)$ is an independent component.

This satisfies truth membership, indeterminacy membership, and falsity membership functions defined as:

$$0\leq\tilde{b}_{\tilde{A}}^{U}(x)+\tilde{s}_{\tilde{A}}^{U}(x)\leq 1 \quad (5)$$

$$0\leq(\tilde{b}_{\tilde{A}}^{U}(x))^{2}+(\tilde{s}_{\tilde{A}}^{U}(x))^{2}\leq 1 \quad (6)$$

$$0\leq\tilde{b}_{\tilde{A}}^{U}(x)+I_{\tilde{A}}^{U}(x)+\tilde{s}_{\tilde{A}}^{U}(x)\leq 2, x\in U \quad (7)$$

$$0\leq(\tilde{b}_{\tilde{A}}^{U}(x))^{2}+(I_{\tilde{A}}^{U}(x))^{2}+(\tilde{s}_{\tilde{A}}^{U}(x))^{2}\leq 2 \quad (8)$$

For every $x\in X$, $\tilde{\pi}_{\tilde{A}}(x)=[\tilde{\pi}_{\tilde{A}}^{U}(x),\tilde{\pi}_{\tilde{A}}^{L}(x)]$ is called IVPNS index of x to $\tilde{P}$,where $\tilde{\pi}_{\tilde{A}}^{U}(x)=\sqrt{1-\tilde{b}_{\tilde{A}}^{U}(x))^{2}-(I_{\tilde{A}}^{U}(x))^{2}-(\tilde{s}_{\tilde{A}}^{U}(x))^{2}}$, $\tilde{\pi}_{\tilde{A}}^{L}(x)=\sqrt{1-\tilde{b}_{\tilde{A}}^{L}(x))^{2}-(I_{\tilde{A}}^{L}(x))^{2}-(\tilde{s}_{\tilde{A}}^{L}(x))^{2}}$.

2.4 Development of IVPNS - COBRA

This section of this research describes the development of the COBRA framework for IVPNS. This study constructs a new linguistic variable for IVPNS according to the 5-Scale and 7-Scale of linguistics variables. In this study, IVPNS COBRA were constructed, and some changes have been made to COBRA without losing the originality of the COBRA method. The overall structure of this proposed approach is shown in the accompanying Figure 1.

The study introduces a new linguistic variable for IVPNS-COBRA and incorporate with identifying the normalization of IVPNS for benefit and cost criteria also of identifying the distance measure using Euclidean and hamming distance for IVPNS approach. The IVPNS-COBRA, which able the determination degree of membership in terms of interval values lower and upper interval that satisfies truth membership $(\tilde{b}_{ij}^{l},\tilde{b}_{ij}^{u})$,the lower and upper interval of indeterminacy membership $(\tilde{I}_{ij}^{l},\tilde{I}_{ij}^{u})$,and lower and upper interval that satisfies falsity membership $(\tilde{s}_{ij}^{l},\tilde{s}_{ij}^{u})$. In this approach, the linguistic variable is derived from the interval Neutrosophic set by Al-Quran *et al.,* [47]. The use of the IVPNS is combination of IVPS and IVNS where both intervals can offer valuable tools addressing uncertainty and complexity in different fields, where IVNS is generally more flexible with indeterminacy, while IVPNS provide a broader and more precise spectrum of truth and falsity, allowing for advanced modelling and decision making.

In this study the Likert scale 5-scale according to Al-Quran *et al.,* [47] was used and modified in terms of IVPNS and at the same time we have modified the 5-likert scale to the 7-Scale in terms of IVPNS. This modification on a 7-likert scale of IVPNS to provide more response options allowing respondents to better express nuanced opinions and capture a broader range of responses. These 5-point and 7-point Likert scales improve the precision and reliability of the assessment process, providing robust data for the COBRA - IVPNS project.

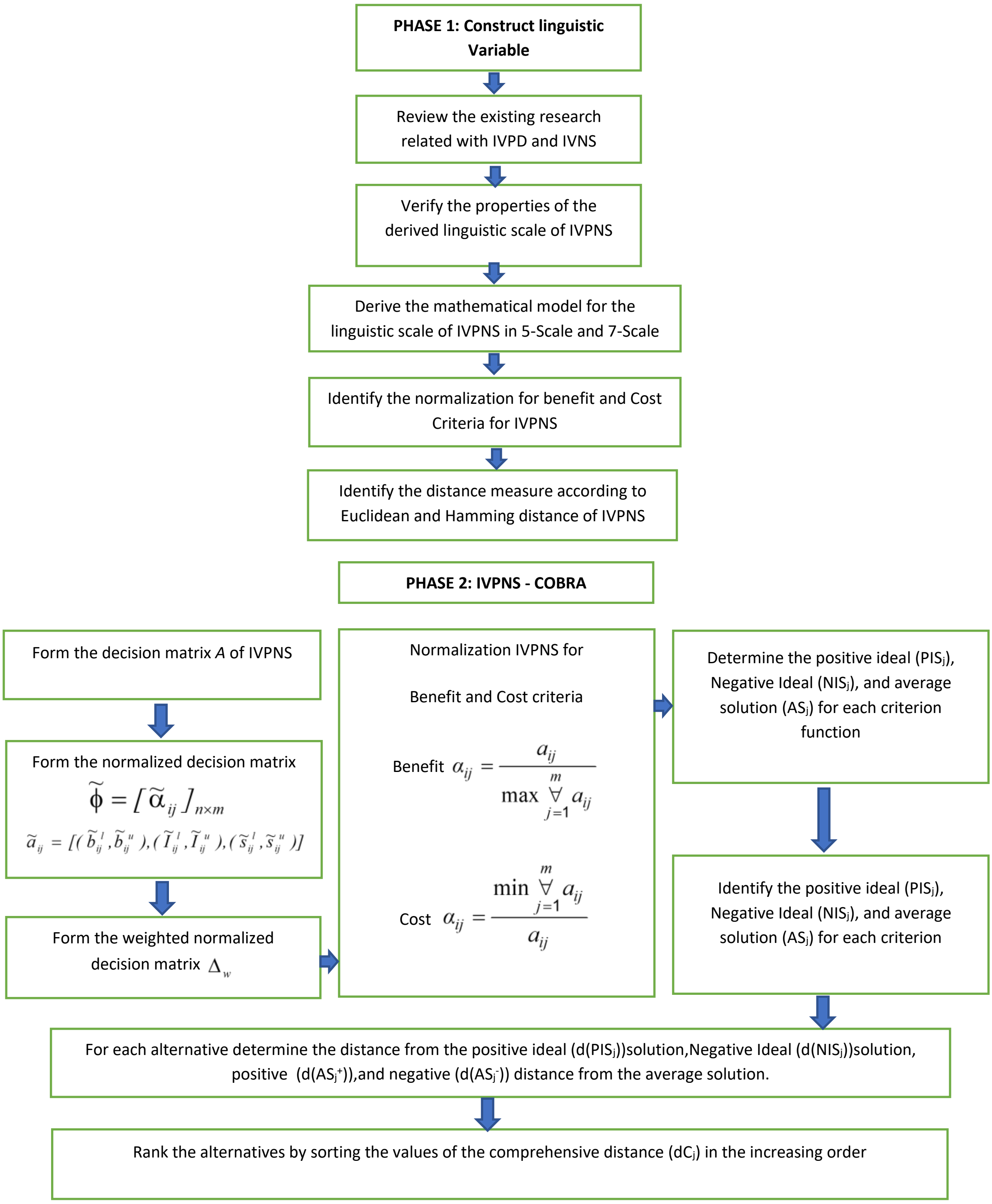

Fig. 1. Proposed IVPNS – COBRA framework

2.5 Construction of Linguistic Variable

The concept of linguistic variables in fuzzy sets has been explored in various ways. According to [44], he introduced the idea of linguistic variables for importance, defining their values as fuzzy subsets and demonstrating how to modify these subsets. The prevalence of degree scales in linguistic variables varies, with the 3-degree and 4-degree scales frequently employed by Rahim *et al.*, [48], although the 5-degree scale is the predominant choice, while scales, exceeding 5 degrees also find utility in certain contexts. The linguistic variable under the Neutrosophic environment and INVS has been developed by [49-51]. In 2021, [52] was introduced the concept of the Linguistic Single-Valued Neutrosophic Soft (LSVNS) set and applied it to game theory. In this investigation, we have constructed a novel linguistic variable tailored to IVPNSs, based on the IVPNS definition. The linguistic variable comprises 5-degree scales and 7-degree scales as outlined in Table 1 and Table 2.

Table 1
The new five linguistic variables under the (IVPNS) concept

Linguistic Variable	Interval Neutrosophic Set
No Influence (NI)	$[0.05, 0.10]$, $[0.70, 0.80]$, $[0.70, 0.90]$
Very Low Influence (VLI)	$[0.20, 0.40]$, $[0.60, 0.70]$, $[0.50, 0.60]$
Medium Influence (MI)	$[0.45, 0.60]$, $[0.40, 0.50]$, $[0.30, 0.40]$
High Influence (HI)	$[0.60, 0.70]$, $[0.30, 0.40]$, $[0.20, 0.30]$
Absolutely Influence (AI)	$[0.80, 0.90]$, $[0.20, 0.30]$, $[0.05, 0.10]$

Table 1, showed the proposed linguistic variable for IVPNS, is arranged on a 5-degree scale. The “No Influence” category represents truth value indicating very low influence, high indeterminacy value and high falsity value indicating the certainty of no influence. The “Very Low Influence” level shows a slight increase in truth while decreasing the indeterminacy and falsity, “Medium High Influence “, “High Influence”, this means the truth value is higher while indeterminacy and falsity are lower, and “Absolutely Influence,” scale is representing a high level of truth with minimal and false respectively. The five linguistic variables satisfied all the conditions according to the definition in IVPNS.

Table 2
The new seven linguistic variables under the IVPNS concept

Linguistic Variable	Interval Pythagorean Neutrosophic Set
No Influence (NI)	$[0.10, 0.15]$, $[0.55, 0.60]$, $[0.65, 0.75]$
Low Influence (LI)	$[0.20, 0.30]$, $[0.50, 0.55]$, $[0.60, 0.65]$
Very Low Influence (VLI)	$[0.35, 0.40]$, $[0.45, 0.50]$, $[0.50, 0.60]$
Medium Influence (MI)	$[0.40, 0.50]$, $[0.30, 0.40]$, $[0.40, 0.45]$
Medium High Influence (MHI)	$[0.50, 0.60]$, $[0.30, 0.40]$, $[0.30, 0.40]$
High Influence (HI)	$[0.60, 0.70]$, $[0.20, 0.30]$, $[0.20, 0.25]$
Absolutely Influence (AI)	$[0.75, 0.85]$, $[0.10, 0.20]$, $[0.10, 0.15]$

In Table 2, the proposed 7-degree scale consists of seven distinct linguistic terms to describe the level of influence. This scale is designed to represent a wide range of influence levels, from minimal to maximal. The terms included in the seven scales representing “No Influence” means suggest a complete absence of influence, “Low Influence”, represents a noticeable but still minor level influence, “Very Low Influence”, implies a slight impact but almost negligible “Medium High Influence “, indicating that an average or balanced level of influence, “Medium High Influence “, suggest an influence level above average but not at the highest levels, “High Influence”, represent a significant level of influence, and “Absolutely Influence,” means indicate the maximum or absolute level of influence respectively. All these seven linguistic characteristics need to comply with specific conditions to satisfy the definition of an IVPNS. Each term in this scale in Table 2 represented as an IVPNS set, with defined interval for truth (B), indeterminacy, (I), and Falsity (S) that satisfied all the conditions according to the definition in IVPNS. This scale provides a way to categorize and analyze influence with more nuance than binary or strictly numerical systems.

2.6 Verify the linguistic variables of IVPNS

The new 5-Likert scale of linguistic variables proposed in Table 1 is verified using all the conditions stated in (5), (6), (7), and (8). Using the definition IVPNS, let $X=\{x_1,x_2,x_3,x_4,x_5\}$ then,

$$A=\begin{Bmatrix} \langle x_1,[0.05,0.10],[0.70,0.80],[0.70,0.90]\ \rangle \\ \langle x_2,[0.20,0.40],[0.60,0.70],[0.50,0.60]\rangle \\ \langle x_3,[0.45,0.60],[0.40,0.50],[0.30,0.40]\rangle \\ \langle x_4,[0.60,0.70],[0.30,0.40],[0.20,0.30]\rangle \\ \langle x_5,[0.80,0.90],[0.20,0.30],[0.05,0.10]\rangle \end{Bmatrix}$$

Is an IVPNS-subset of X where $\tilde{b},I,\tilde{s}\in[0,1]$. Using the definition of the IVPNS in Equation (4), we have $\tilde{b}_{\tilde{A}}(x)=[\tilde{b}^L_{\tilde{A}}(x),\tilde{b}^U_{\tilde{A}}(x)]\subseteq[0,1],\quad I_{\tilde{A}}(x)=[I^L_{\tilde{A}}(x),I^U_{\tilde{A}}(x)]\subseteq[0,1],$ and $s_{\tilde{A}}(x)=[s^L_{\tilde{A}}(x),s^U_{\tilde{A}}(x)]\subseteq[0,1]$. Therefore, these numbers comply with the condition $[0.20,0.40],[0.60,0.70],[0.50,0.60]\in[0,1]$. To validate the suggested interval value, we investigate the conditions in (5), (6), (7), and (8) using the following ways:

Let $A_1=[0.20,0.40],[0.60,0.70],[0.50,0.60]\in[0,1]$ and $A_2=[0.45,0.60],[0.40,0.50],[0.30,0.40]\in[0,1]$ is the IVPNS numbers. The condition in equations (5), (6), (7), and (8) are checked in the manner described below:

i) $\tilde{b}^U_{\tilde{A}}(x)+\tilde{s}^U_{\tilde{A}}(x)\le 1$

$\tilde{b}^U_{\tilde{A}_1}(x)+\tilde{s}^U_{\tilde{A}_1}(x)=0.40+0.60=1\le 1$

$\tilde{b}^U_{\tilde{A}_2}(x)+\tilde{s}^U_{\tilde{A}_2}(x)=0.60+0.40=1\le 1$

Satisfied Equation (5)

ii) $[\tilde{b}^U_{\tilde{A}}(x)]^2+[\tilde{s}^U_{\tilde{A}}(x)]^2\le 1$

$[\tilde{b}^U_{\tilde{A}_1}(x)]^2+[\tilde{s}^U_{\tilde{A}_1}(x)]^2=0.40^2+0.60^2=0.52\le 1$

$[\tilde{b}^U_{\tilde{A}_2}(x)]^2+[\tilde{s}^U_{\tilde{A}_2}(x)]^2=0.60^2+0.40^2=0.52\le 1$

Satisfied Equation (6)

iii) $0\le\tilde{b}^U_{\tilde{A}}(x)+I^U_{\tilde{A}}(x)+\tilde{s}^U_{\tilde{A}}(x)\le 2, x\in U$

$\tilde{b}^U_{\tilde{A}_1}(x)+I^U_{\tilde{A}_1}(x)+\tilde{s}^U_{\tilde{A}_1}(x)=0.40+0.70+0.60=1.70$

$\tilde{b}^U_{\tilde{A}_2}(x)+I^U_{\tilde{A}_2}(x)+\tilde{s}^U_{\tilde{A}_2}(x)=0.60+0.50+0.40=1.50$

Satisfied Equation (7)

iv) $[\tilde{b}^U_{\tilde{A}}(x)]^2+[I^U_{\tilde{A}}(x)]^2+[\tilde{s}^U_{\tilde{A}}(x)]^2\le 2$

$[\tilde{b}^U_{A_1}(x)]^2+[I^U_{A_1}(x)]^2+[\tilde{s}^U_{A_1}(x)]^2=0.40^2+0.70^2+0.60^2=1.01$

$[\tilde{b}^U_{A_2}(x)]^2+[I^U_{A_2}(x)]^2+[\tilde{s}^U_{A_2}(x)]^2=0.60^2+0.50^2+0.40^2=0.77$

Satisfied Equation (8)

Table 3 below shows the verification for each defined value of IVPNS using the 5-Likert scale, where each set satisfies all the 4-condition stated in equation (5), (6), (7), and (8).

Table 3
Verified linguistic variable of IVPNS for 5-Likert scale

$\tilde{b}$	$\tilde{I}$	$\tilde{s}$	$\tilde{b}^U + \tilde{s}^U \leq 1$	$(\tilde{b}^U)^2 + (\tilde{s}^U)^2 \leq 1$	$0 \leq \tilde{b}^U + \tilde{I}^2 + \tilde{s}^U \leq 2$	$0 \leq (\tilde{b}^U)^2 + (\tilde{I}^U)^2 + (\tilde{s}^U)^2 \leq 2$
[0.00,0.10]	[0.70,0.80]	[0.70,0.90]	1.0	0.82	1.80	1.46
[0.20,0.40]	[0.60,0.70]	[0.50,0.60]	1.0	0.52	1.70	1.01
[0.45,0.60]	[0.40,0.50]	[0.30,0.40]	1.0	0.52	1.50	0.77
[0.60,0.70]	[0.30,0.40]	[0.20,0.30]	1.0	0.58	1.40	0.74
[0.80,0.90]	[0.20,0.30]	[0.05,0.10]	1.0	0.82	1.30	0.91

The new 7-Likert scale of linguistic variables proposed in Table 2 is verified using all the conditions stated using equation (5), (6), (7), (8). Using the definition IVPNS, let $X = \{x_1, x_2, x_3, x_4, x_5\}$ then

$$A = \begin{Bmatrix} \langle x,[0.10,0.15],[0.55,0.60],[0.65,0.75]\rangle \\ \langle x,[0.20,0.30],[0.50,0.55],[0.60,0.65]\rangle \\ \langle x,[0.35,0.40],[0.45,0.50],[0.50,0.60]\rangle \\ \langle x,[0.40,0.50],[0.30,0.40],[0.40,0.45]\rangle \\ \langle x,[0.50,0.60],[0.30,0.40],[0.30,0.40]\rangle \\ \langle x,[0.60,0.70],[0.20,0.30],[0.20,025]\rangle \\ \langle x,[0.75,0.85],[0.10,0.20],[0.10,0.15]\rangle \end{Bmatrix}$$

Is an IVPNS-subset of X where $\tilde{b}, I, \tilde{s} \in [0,1]$. Using the definition of the IVPNS in Equation (4), we have $\tilde{b}_{\tilde{A}}(x) = [\tilde{b}^L_{\tilde{A}}(x), \tilde{b}^U_{\tilde{A}}(x)] \subseteq [0,1],\ I_{\tilde{A}}(x) = [I^L_{\tilde{A}}(x), I^U_{\tilde{A}}(x)] \subseteq [0,1],$ and $s_{\tilde{A}}(x) = [s^L_{\tilde{A}}(x), s^U_{\tilde{A}}(x)] \subseteq [0,1]$. Therefore, these numbers comply with the condition $[0.10,0.15],[0.55,0.60],[0.65,0.75] \subseteq [0,1]$. To support and validate the suggested interval value, the conditions in equation (5), (6), (7), and (8) are investigated in the following ways.

Let $A_1 = \{\langle [0.10,0.15],[0.55,0.60],[0.65,0.75]\rangle\}$ and $A_2 \{\langle [0.20,0.30],[0.50,0.55],[0.60,0.65]\rangle\}$ is the IVPNS numbers. The condition (5), (6), (7), and (8) are verified in the manner described below:

i) $\tilde{b}^U_{\tilde{A}}(x) + \tilde{s}^U_{\tilde{A}} \leq 1$

$\tilde{b}^U_{A_1}(x) + \tilde{s}^U_{A_1} = 0.15 + 0.75 = 0.90 \leq 1$

$\tilde{b}^U_{A_2}(x) + \tilde{s}^U_{A_2} = 0.30 + 0.65 = 0.95 \leq 1$

Satisfied Equation (5)

ii) $[\tilde{b}^U_{\tilde{A}}(x)]^2 + [\tilde{s}^U_{\tilde{A}}(x)]^2 \leq 1$

$[\tilde{b}^U_{\tilde{A}_1}(x)]^2 + [\tilde{s}^U_{\tilde{A}_1}(x)]^2 = 0.15^2 + 0.75^2 = 0.5625 \leq 1$

$[\tilde{b}^U_{\tilde{A}_2}(x)]^2 + [\tilde{s}^U_{\tilde{A}_2}(x)]^2 = 0.30^2 + 0.65^2 = 0.5125 \leq 1$

Satisfied Equation (6)

iii) $0 \leq \tilde{b}^U_{\tilde{A}}(x) + I^U_{\tilde{A}}(x) + \tilde{s}^U_{\tilde{A}}(x) \leq 2, x \in U$

$\tilde{b}^U_{\tilde{A}_1}(x) + I^U_{\tilde{A}_1}(x) + \tilde{s}^U_{\tilde{A}_1}(x) = 0.15 + 0.60 + 0.75 = 1.50$

$\tilde{b}^U_{\tilde{A}_2}(x) + I^U_{\tilde{A}_2}(x) + \tilde{s}^U_{\tilde{A}_2}(x) = 0.30 + 0.55 + 0.65 = 1.50$

Satisfied Equation (7)

iv) $[\tilde{b}^U_{\tilde{A}}(x)]^2 + [I^U_{\tilde{A}}(x)]^2 + [\tilde{s}^U_{\tilde{A}}(x)]^2 \leq 2$

$[\tilde{b}^U_{A_1}(x)]^2 + [I^U_{A_1}(x)]^2 + [\tilde{s}^U_{A_1}(x)]^2 = 0.15^2 + 0.60^2 + 0.75^2 = 0.945$

$[\tilde{b}^U_{A_2}(x)]^2 + [I^U_{A_2}(x)]^2 + [\tilde{s}^U_{A_2}(x)]^2 = 0.30^2 + 0.55^2 + 0.65^2 = 0.815$

Satisfied Equation (8)

Table 4 below shows the verification for each defined value of IVPNS using 7-likerts scale, where each set satisfies all the 4 conditions stated in (5), (6), (7), and (8).

Table 4
Verified linguistic variable of IVPNS for the 7-Likert scale

$\tilde{b}$	$\tilde{I}$	$\tilde{s}$	$\tilde{b}^U + \tilde{s}^U \leq 1$	$(\tilde{b}^U)^2 + (\tilde{s}^U)^2 \leq 1$	$0 \leq \tilde{b}^U + \tilde{I}^2 + \tilde{s}^U \leq 2$	$0 \leq (\tilde{b}^U)^2 + (\tilde{I}^U)^2 + (\tilde{s}^U)^2 \leq 2$
[0.10,0.15]	[0.55,0.60]	[0.65,0.75]	0.9	0.585	1.50	0.945
[0.20,0.30]	[0.50,0.55]	[0.60,0.65]	0.95	0.513	1.50	0.815
[0.35,0.40]	[0.45,0.50]	[0.50,0.60]	1.0	0.52	1.50	0.77
[0.40,0.50]	[0.30,0.40]	[0.40,0.45]	0.95	0.453	1.45	0.613
[0.50,0.60]	[0.30,0.40]	[0.30,0.40]	1.0	0.52	1.40	0.68
[0.60,0.70]	[0.20,0.30]	[0.20,0.25]	0.95	0.553	1.25	0.643
[0.75,0.85]	[0.10,0.20]	[0.10,0.15]	1.0	0.745	1.20	0.785

3. Methodology

In this section, the normalization process for IVPNS is defined. This study introduces a new normalization approach to improve the accuracy and reliability of decision-making models, particularly in MCDM problems. The proposed normalization method is applied to both benefit and cost criteria, ensuring that benefit attributes are maximized, while cost attributes are minimized. This approach enhances the comparability of different criteria, allowing for a more balanced and precise evaluation of alternatives under uncertainty.

3.1 Normalization of IVPNS

Definition 4. let $\tilde{\varphi} = [\tilde{\alpha}_{ij}]_{n\times m}$, where, $\tilde{a}_{ij} = [(\tilde{b}^l_{ij}, \tilde{b}^u_{ij}), (\tilde{I}^l_{ij}, \tilde{I}^u_{ij}), (\tilde{s}^l_{ij}, \tilde{s}^u_{ij})]$ is a normalized IVPNS, whose lower and upper intervals that satisfy truth membership $(\tilde{b}^l_{ij}, \tilde{b}^u_{ij})$, a lower and upper interval that satisfies indeterminacy membership $(\tilde{I}^l_{ij}, \tilde{I}^u_{ij})$, and lower and upper interval that satisfies falsity membership $(\tilde{s}^l_{ij}, \tilde{s}^u_{ij})$ values are obtained as follows:

1. Benefit Criteria $\alpha_{ij} = \dfrac{a_{ij}}{\max \overset{m}{\underset{j=1}{\forall}} a_{ij}}$, $\tilde{a}_{ij} = [(\tilde{b}^l_{ij}, \tilde{b}^u_{ij}), (\tilde{I}^l_{ij}, \tilde{I}^u_{ij}), (\tilde{s}^l_{ij}, \tilde{s}^u_{ij})]$

$$(\tilde{b}^l_{ij}, \tilde{b}^u_{ij}) = \left(\frac{b^l_{ij}}{\max\limits_{j=1}^{m}(b^l_{ij}, b^u_{ij})}, \frac{b^u_{ij}}{\max\limits_{j=1}^{m}(b^l_{ij}, b^u_{ij})} \right), (\tilde{I}^l_{ij}, \tilde{I}^u_{ij}) = \left(\frac{I^l_{ij}}{\max\limits_{j=1}^{m}(I^l_{ij}, I^u_{ij})}, \frac{I^u_{ij}}{\max\limits_{j=1}^{m}(I^l_{ij}, I^u_{ij})} \right)$$

$$(\tilde{s}^l_{ij}, \tilde{s}^u_{ij}) = \left(\frac{s^l_{ij}}{\max\limits_{j=1}^{m}(s^l_{ij}, s^u_{ij})}, \frac{s^u_{ij}}{\max\limits_{j=1}^{m}(s^l_{ij}, s^u_{ij})} \right) \tag{9}$$

2. Cost Criteria $\alpha_{ij} = \dfrac{\min \overset{m}{\underset{j=1}{\forall}} a_{ij}}{a_{ij}}$, $\tilde{a}_{ij} = [(\tilde{b}^u_{ij}, \tilde{b}^l_{ij}), (\tilde{I}^u_{ij}, \tilde{I}^l_{ij}), (\tilde{s}^u_{ij}, \tilde{s}^l_{ij})]$

$$(\tilde{b}_{ij}^{l},\tilde{b}_{ij}^{u})=\left(\frac{\min\limits_{j=1}^{m}(b_{ij}^{l},b_{ij}^{u})}{b_{ij}^{u}},\frac{\min\limits_{j=1}^{m}(b_{ij}^{l},b_{ij}^{u})}{b_{ij}^{l}}\right),(\tilde{I}_{ij}^{l},\tilde{I}_{ij}^{u})=\left(\frac{\min\limits_{j=1}^{m}(I_{ij}^{l},I_{ij}^{u})}{I_{ij}^{u}},\frac{\min\limits_{j=1}^{m}(I_{ij}^{l},I_{ij}^{u})}{I_{ij}^{l}}\right)$$

$$(\tilde{s}_{ij}^{l},\tilde{s}_{ij}^{u})=\left(\frac{\min\limits_{j=1}^{m}(s_{ij}^{l},s_{ij}^{u})}{s_{ij}^{u}},\frac{\min\limits_{j=1}^{m}(s_{ij}^{l},s_{ij}^{u})}{s_{ij}^{l}}\right) \tag{10}$$

3.1 Distance Measure For IVPNS

In this section, we define the Hamming and Euclidean distances for the IVPNS.

Definition 5. Let U be a universe of discourse (objects) with generic elements in U denote by x. Then the IVPNS $\tilde{n}$ is an object having the for

$$\tilde{n}=\{<x,[\tilde{b}_{\tilde{n}}^{L},\tilde{b}_{\tilde{n}}^{U}],\tilde{I}_{\tilde{n}}^{L},\tilde{I}_{\tilde{n}}^{U}],[\tilde{s}_{\tilde{n}}^{L},\tilde{s}_{\tilde{n}}^{U}]\big|x\in X>\}$$

Then we define the following Euclidian and Hamming distances for IVPNS as follows:
Where $dE(S_j)_i$ and $dH(S_j)_i$, represent the Euclidian and Hamming distances respectively, which are the calculation for the positive ideal solutions obtained in the following way:

$$dE(PIS_j)_i=\left\{\frac{1}{6}\sum_{j=1}^{m}(\left|PIS_j-(w_j\times\tilde{b}_{ij}^{l})\right|^2+\left|PIS_j-(w_j\times\tilde{b}_{ij}^{u})\right|^2+\left|PIS_j-(w_j\times\tilde{I}_{ij}^{l})\right|^2+\left|PIS_j-(w_j\times\tilde{I}_{ij}^{u})\right|^2\right.$$
$$\left.\left|PIS_j-(w_j\times\tilde{I}_{ij}^{l})\right|^2+\left|PIS_j-(w_j\times\tilde{I}_{ij}^{u})\right|^2)\right\}\quad\forall_i=1,2,\dots,n,\forall_j=1,2,\dots,m, \tag{11}$$

$$dH(PIS_j)_i=\left\{\frac{1}{6}\sum_{j=1}^{m}(\left|PIS_j-(w_j\times\tilde{b}_{ij}^{l})\right|+\left|PIS_j-(w_j\times\tilde{b}_{ij}^{u})\right|+\left|PIS_j-(w_j\times\tilde{I}_{ij}^{l})\right|+\left|PIS_j-(w_j\times\tilde{I}_{ij}^{u})\right|\right.$$
$$\left.\left|PIS_j-(w_j\times\tilde{I}_{ij}^{l})\right|+\left|PIS_j-(w_j\times\tilde{I}_{ij}^{u})\right|)\right\}\quad\forall_i=1,2,\dots,n,\forall_j=1,2,\dots,m, \tag{12}$$

For the negative ideal solutions, the Euclidian and Hamming distances are obtained in the following way:

$$dE(NIS_j)_i=\left\{\frac{1}{6}\sum_{j=1}^{m}(\left|NIS_j-(w_j\times\tilde{b}_{ij}^{l})\right|^2+\left|NIS_j-(w_j\times\tilde{b}_{ij}^{u})\right|^2+\left|NIS_j-(w_j\times\tilde{I}_{ij}^{l})\right|^2+\left|NIS_j-(w_j\times\tilde{I}_{ij}^{u})\right|^2\right.$$
$$\left.\left|NIS_j-(w_j\times\tilde{I}_{ij}^{l})\right|^2+\left|NIS_j-(w_j\times\tilde{I}_{ij}^{u})\right|^2)\right\}\ \forall_i=1,2,\dots,n,\forall_j=1,2,\dots,m, \tag{13}$$

$$dH(NIS_j)_i=\left\{\frac{1}{6}\sum_{j=1}^{m}(\left|NIS_j-(w_j\times\tilde{b}_{ij}^{l})\right|+\left|NIS_j-(w_j\times\tilde{b}_{ij}^{u})\right|+\left|NIS_j-(w_j\times\tilde{I}_{ij}^{l})\right|+\left|NIS_j-(w_j\times\tilde{I}_{ij}^{u})\right|\right.$$
$$\left.\left|NIS_j-(w_j\times\tilde{I}_{ij}^{l})\right|+\left|NIS_j-(w_j\times\tilde{I}_{ij}^{u})\right|)\right\}\quad\forall_i=1,2,\dots,n,\forall_j=1,2,\dots,m. \tag{14}$$

For the positive distance from the average solutions, the Euclidian and Hamming distances are calculated as follows:

$$dE(AS_j)_i=\left\{\frac{1}{6}\sum_{j=1}^{m}(\tau^+\left|AS_j-(w_j\times\tilde{b}_{ij}^{l})\right|^2+\tau^+\left|AS_j-(w_j\times\tilde{b}_{ij}^{u})\right|^2+\tau^+\left|AS_j-(w_j\times\tilde{I}_{ij}^{l})\right|^2+\tau^+\left|AS_j-(w_j\times\tilde{I}_{ij}^{u})\right|^2\right.$$
$$\left.\tau^+\left|AS_j-(w_j\times\tilde{I}_{ij}^{l})\right|^2+\tau^+\left|AS_j-(w_j\times\tilde{I}_{ij}^{u})\right|^2)\right\}\ \forall_i=1,2,\dots,n,\forall_j=1,2,\dots,m. \tag{15}$$

$$dH(AS_j)_i=\left\{\frac{1}{6}\sum_{j=1}^{m}(\tau^+\left|AS_j-(w_j\times\tilde{b}_{ij}^{l})\right|+\tau^+\left|AS_j-(w_j\times\tilde{b}_{ij}^{u})\right|+\tau^+\left|AS_j-(w_j\times\tilde{I}_{ij}^{l})\right|+\tau^+\left|AS_j-(w_j\times\tilde{I}_{ij}^{u})\right|\right.$$
$$\left.\tau^+\left|AS_j-(w_j\times\tilde{I}_{ij}^{l})\right|+\tau^+\left|AS_j-(w_j\times\tilde{I}_{ij}^{u})\right|)\right\}\ \forall_i=1,2,\dots,n,\forall_j=1,2,\dots,m. \tag{16}$$

$$\tau^{+}\begin{cases}1 \; if \;\; AS_j < w_j \times a_{ij}\\ 0 \; if \;\; AS_j \geq w_j \times a_{ij}\end{cases}$$

This section of the condition for ensuring positive distance from the average solutions, as identified by Kristic *et al.*, [43] presents a limitation. We modified the second condition, transitioning it from $\tau^{+}=0 \; if \;\; AS > w_j \times a_{ij}$ to $\tau^{+}=0 \; if \;\; AS \geq w_j \times a_{ij}$.

For the negative distance from the average solutions, the Euclidian and Hamming distances are calculated in the following manner:

$$dE(AS_j)_i = \left\{ \frac{1}{6} \sum_{j=1}^{m} (\tau^{-} \left| AS_j - (w_j \times \tilde{b}_{ij}^{l}) \right|^2 + \tau^{-} \left| AS_j - (w_j \times \tilde{b}_{ij}^{u}) \right|^2 + \tau^{-} \left| AS_j - (w_j \times \tilde{I}_{ij}^{l}) \right|^2 + \tau^{-} \left| AS_j - (w_j \times \tilde{I}_{ij}^{u}) \right|^2 \right.$$
$$\left. \tau^{-} \left| AS_j - (w_j \times \tilde{I}_{ij}^{l}) \right|^2 + \tau^{-} \left| AS_j - (w_j \times \tilde{I}_{ij}^{u}) \right|^2) \right\} \; \forall_i = 1,2,\ldots,n, \forall_j = 1,2,\ldots,m. \quad (17)$$

$$dH(AS_j)_i = \left\{ \frac{1}{6} \sum_{j=1}^{m} (\tau^{-} \left| AS_j - (w_j \times \tilde{b}_{ij}^{l}) \right| + \tau^{-} \left| AS_j - (w_j \times \tilde{b}_{ij}^{u}) \right| + \tau^{-} \left| AS_j - (w_j \times \tilde{I}_{ij}^{l}) \right| + \tau^{-} \left| AS_j - (w_j \times \tilde{I}_{ij}^{u}) \right| \right.$$
$$\left. \tau^{-} \left| AS_j - (w_j \times \tilde{I}_{ij}^{l}) \right| + \tau^{-} \left| AS_j - (w_j \times \tilde{I}_{ij}^{u}) \right|) \right\} \; \forall_i = 1,2,\ldots,n, \forall_j = 1,2,\ldots,m. \quad (18)$$

$$\tau^{-}\begin{cases}1 \; if \;\; AS_j > w_j \times a_{ij}\\ 0 \; if \;\; AS_j \leq w_j \times a_{ij}\end{cases},$$

This section highlights a limitation related to ensuring a negative distance from the average solutions, as identified by Kristic *et al.,* [43]. We modified the second condition, transitioning it from $\tau^{+}=0 \; if \;\; AS_j < w_j \times a_{ij}$ to $\tau^{+}=0 \; if \;\; AS_j \leq w_j \times a_{ij}$.

3.2 The IVPNS -COBRA Procedure

The COBRA method was established by Krstic *et al.,* [43], representing a distance based MCDM approach. It ranks alternatives by integrating two types of distances of the alternatives, namely Euclidian and Taxicab. The newly established COBRA method, used in this study to obtain the final ranking of the alternatives, consists of several sub-steps:

Step 1. Establish the evaluations a_{ij} if the alternatives $i,(i=1,2,3,\ldots,n)$ with criteria $j,(j=1,2,3,\ldots,n)$, thus forming the decision matrix A:

$$A = \left[\begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \right]$$

Where $a_{ij} = [(b_{ij}^{l}, b_{ij}^{u}), (I_{ij}^{l}, I_{ij}^{u}), (s_{ij}^{l}, s_{ij}^{u})]$ are the evaluation of the alternatives i with criteria j obtain using the scale in IVPNS. The n is the total number of alternatives, m is the total number of criteria taken into consideration and $(\tilde{b}_{ij}^{l}, \tilde{b}_{ij}^{u}), (\tilde{I}_{ij}^{l}, \tilde{I}_{ij}^{u}), (\tilde{s}_{ij}^{l}, \tilde{s}_{ij}^{u})$ the lower and upper interval that satisfies truth membership, indeterminacy membership, and falsity membership, respectively.

Step 2. Form the normalized IVPNS decision matrix $\tilde{\varphi}$ by using equation (9) for Cost criteria and equation (10) for Benefit criteria.

Step 3. Form the weighted normalized decision matrix Δ_w in the following way:

$$\tilde{\varphi} = [w_{ij} \times \tilde{\alpha}_{ij}]_{n \times m} \quad (19)$$

Where w_j denote the relative weight of criterion j.

Step 4. For each criterion function determine the positive ideal (PIS_j), negative ideal (NIS_j), and average ideal (AS_j) in the following way:

$$PIS_j = (\tilde{b}_{ij}^l, \times \tilde{b}_{ij}^u)^{PIS}$$
$$= [\max_j (w_j \times \tilde{b}_{ij}^l), \max_j (w_j \times \tilde{b}_{ij}^u)]$$
$$PIS_j = [(\tilde{I}_{ij}^l, \tilde{I}_{ij}^u)^{PIS}, (\tilde{s}_{ij}^l, \tilde{s}_{ij}^u)^{PIS}]$$
$$= [\min(w_j \times \tilde{I}_{ij}^l), \min_j (w_j \times \tilde{I}_{ij}^u)], [\min_j (w_j \times \tilde{s}_{ij}^l), \min_j (w_j \times \tilde{s}_{ij}^u)]$$
$$\forall j = 1, 2, 3, \dots, m \ for \ j \in B, C \tag{20}$$
$$NIS_j = (\tilde{b}_{ij}^l, \times \tilde{b}_{ij}^u)^{NIS}$$
$$= [\min_j (w_j \times \tilde{b}_{ij}^l), \min_j (w_j \times \tilde{b}_{ij}^u)]$$
$$NIS_j = [(\tilde{I}_{ij}^l, \tilde{I}_{ij}^u)^{NIS}, (\tilde{s}_{ij}^l, \tilde{s}_{ij}^u)^{NIS}]$$
$$= [\max(w_j \times \tilde{I}_{ij}^l), \max_j (w_j \times \tilde{I}_{ij}^u)], [\max_j (w_j \times \tilde{s}_{ij}^l), \max_j (w_j \times \tilde{s}_{ij}^u)]$$
$$\forall j = 1, 2, 3, \dots, m \ for \ j \in B, C \tag{21}$$
$$AS_j = [(\tilde{b}_{ij}^l, \times \tilde{b}_{ij}^u)^{AS}, (\tilde{I}_{ij}^l, \tilde{I}_{ij}^u)^{AS}, (\tilde{s}_{ij}^l, \tilde{s}_{ij}^u)^{AS}]$$
$$= [\underset{j}{mean}(w_j \times \tilde{b}_{ij}^l), \underset{j}{mean}(w_j \times \tilde{b}_{ij}^u)], [\underset{j}{mean}(w_j \times \tilde{I}_{ij}^l), \underset{j}{mean}(w_j \times \tilde{I}_{ij}^u)], [\underset{j}{mean}(w_j \times \tilde{s}_{ij}^l), \underset{j}{mean}(w_j \times \tilde{s}_{ij}^u)]$$
$$\forall j = 1, 2, 3, \dots, m \ for \ j \in B, C \tag{22}$$

where B is the set benefit and C is the set cost criteria.

Step 5. For each alternative determine the distance from the positive ideal $(d(PIS_j))$ and negative ideal $(d(NIS_j))$ the solution should be defined. Also, the positive $(d(AS_j^+))$ and negative $(d(AS_j^-))$ distance from the average solutions should be determined. This procedure is performed in the following way:

$$(d(S_j)) = dE(S_j) + \sigma \times dE(S_j) \times dH(S_j), \forall j = 1, 2, 3, \dots, m \tag{23}$$

Where S_j is any solution $(PIS_{j,} NIS_j, \text{ or } AS_j)$, σ represent the correction coefficient obtained by using the following equation:

$$\sigma = \max_i dE(S_j)_i - \min_i dE(S_j)_i \tag{24}$$

The calculation for the Euclidean and Hamming distance for the positive ideal solutions can be obtained by using equations (11) and (12), while the negative ideal solution is obtained according to equations (13) and (14).

The calculation for the Euclidean and Hamming distance for the positive distance from the average solutions can be obtained in equations (15) and (16) and for the negative distance from the average solutions can be obtained in equations (17) and (18).

Step 6. Rank the considered alternatives in ascending order based on the comprehensive distance which is defined by using:

$$dC_i = \frac{d(PIS_j) - d(NIS_j) - d(AS_j)^+ + d(AS_j)^-}{4}, \forall i = 1, 2, 3, \dots, n \tag{25}$$

4. Illustrative Example

In this section, the proposed model will be illustrated by using 5 and 7-likert scale of IVPNS value to show the applicability of the linguistic variable. The example regarding to the decision making will be solve under IVPNS environment by using COBRA method.

4.1 Five-Likert scale of IVPNS

In this part, the illustrative example of e-commerce development strategies from Karabašević *et al.*, [49] and Popović, [44] were used. We will use a 5-Likert scale of linguistic variable IVPNS to solve the numerical example and demonstrate how applicable it is. Three strategies are being evaluated according to five set criteria involved illustrated in Figure 2.

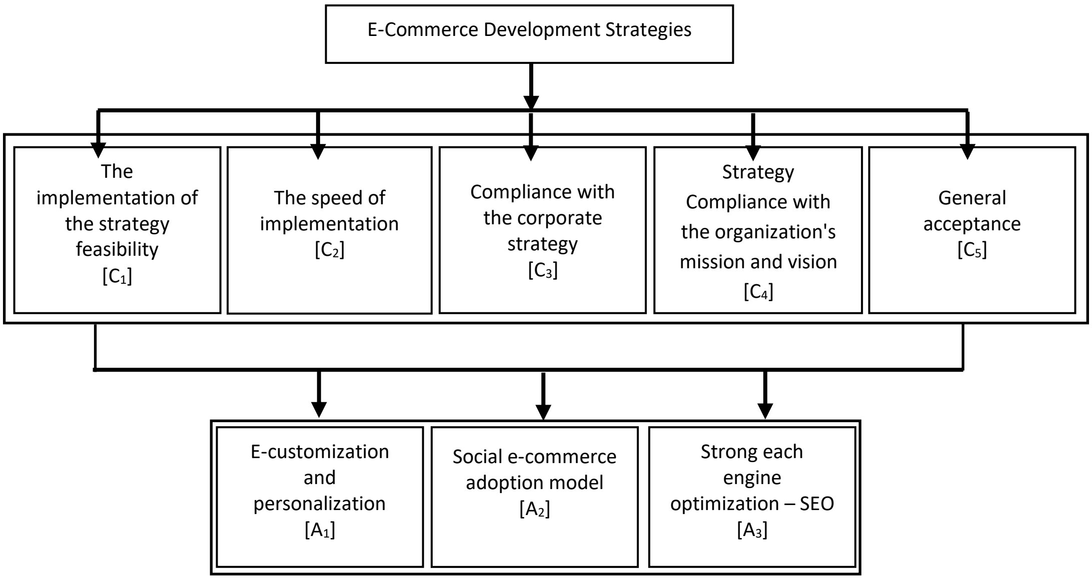

Fig. 2. Selection of E-Commerce Development Strategy

The decision-making for e-commerce development strategies involved only one decision maker and his rating using IVPNS is presented in Table 5 below.

Table 5
Decision-Makers Rating Alternative Strategies

	A_1	A_2	A_3
	[0.45,0.60]	[0.80,0.90]	[0.45,0.60]
C_1	[0.40,0.50]	[0.20,0.30]	[0.40,0.50]
	[0.30,0.40]	[0.05,0.10]	[0.30,0.40]
	[0.45,0.60]	[0.60,0.70]	[0.45,0.60]
C_2	[0.40,0.50]	[0.30,0.40]	[0.40,0.50]
	[0.30,0.40]	[0.20,0.30]	[0.30,0.40]
	[0.45,0.60]	[0.80,0.90]	[0.60,0.70]
C_3	[0.40,0.50]	[0.20,0.30]	[0.30,0.40]
	[0.30,0.40]	[0.05,0.10]	[0.20,0.30]
	[0.20,0.40]	[0.80,0.90]	[0.80,0.90]
C_4	[0.60,0.70]	[0.20,0.30]	[0.20,0.30]
	[0.50,0.60]	[0.05,0.10]	[0.05,0.10]
	[0.20,0.40]	[0.80,0.90]	[0.80,0.90]
C_5	[0.60,0.70]	[0.20,0.30]	[0.20,0.30]
	[0.50,0.60]	[0.05,0.10]	[0.05,0.10]

The weight of decision-making criteria for developing e-commerce strategies is determined according to [44] and is illustrated presented in Table 6 below. This table highlights that the criteria for Strategy Compliance with the organization's mission and vision, along with General acceptance carry the same and highest weight among the criteria considered.

Table 6
The Criteria Weight of E-Commerce Development Strategies

Crit.	C_1	C_2	C_3	C_4	C_5
w_j	0.097	0.056	0.153	0.347	0.347

Table 6 shows the criteria weight value where the Strategy Compliance with the organization's mission and vision (C_4) and General acceptance (C_5) have the same highest value among the other criteria.

Now, to attain the ultimate ranking order of the strategies under consideration, the COBRA-IVPNS method is presently employed.

4.2 COBRA-IVPNS Method

Step 1. Forming decision maker rating of the alternative strategies in Table 5 to the decision matrix A.

Step 2. Form the normalized IVPNS in decision matrix A using Equation (9), all these criteria are Benefit Criteria. Table 7 shown the normalized of COBRA-IVPNS.

Calculation for C_1S_1:

$$b_{C_1A_1} = \max(0.450, 0.600, 0.800, 0.900, 0.450, 0.600) = 0.900$$

$$\frac{0.450}{0.900} = 0.500, \frac{0.600}{0.900} = 0.667$$

$$I_{C_1A_1} = \max(0.400, 0.500, 0.200, 0.300, 0.400, 0.500) = 0.500$$

$$\frac{0.400}{0.500} = 0.800, \frac{0.500}{0.500} = 1.000$$

$$s_{C_1A_1} = \max(0.300, 0.400, 0.050, 0.100, 0.300, 0.400) = 0.400$$

$$\frac{0.300}{0.400} = 0.750, \frac{0.400}{0.400} = 1.000$$

Table 7
The Normalized of COBRA-IVPNS

	C_1	C_2	C_3	C_4	C_5
	[0.500,0.667]	[0.643,0.857]	[0.500,0.667]	[0.222,0.444]	[0.222,0.444]
A_1	[0.800,0.100]	[0.800,1.000]	[0.800,1.000]	[0.857,1.000]	[0.857,1.000]
	[0.750,1.000]	[0.750,1.000]	[0.750,1.000]	[0.833,1.000]	[0.833,1.000]
	[0.889,1.000]	[0.857,1.000]	[0.889,1.000]	[0.889,1.000]	[0.889,1.000]
A_2	[0.400,0.600]	[0.600,0.800]	[0.400,0.600]	[0.286,0.429]	[0.286,0.429]
	[0.125,0.250]	[0.500,0.750]	[0.125,0.250]	[0.083,0.167]	[0.083,0.167]
	[0.500,0.667]	[0.643,0.857]	[0.667,0.778]	[0.889,1.000]	[0.889,1.000]
A_3	[0.800,1.000]	[0.800,1.000]	[0.600,0.800]	[0.286,0.429]	[0.286,0.429]
	[0.750,1.000]	[0.750,1.000]	[0.500,0.750]	[0.083,0.167]	[0.083,0.167]

Step 3. Form the weight of normalized IVPNS in step 2 by using Equation (19).

Calculation for C_1A_1, in this step we multiply the normalized IVPNS with the criteria weight given in Table 6 and the result as shown in Table 8:

$$0.500 \times 0.097 = 0.049, 0.667 \times 0.097 = 0.065$$

$$0.800 \times 0.097 = 0.0078, 1.000 \times 0.097 = 0.097$$

$$0.750 \times 0.097 = 0.073, 0.1.000 \times 0.097 = 0.097$$

Table 8
The Normalized Weight of IVPNS

	C_1	C_2	C_3	C_4	C_5
	[0.049,0.065]	[0.036,0.048]	[0.077,0.102]	[0.077,0.154]	[0.077,0.154]
A_1	[0.078,0.097]	[0.045,0.056]	[0.122,0.153]	[0.297,0.347]	[0.297,0.347]
	[0.073,0.097]	[0.042,0.056]	[0.115,0.153]	[0.289,0.347]	[0.289,0.347]
	[0.086,0.097]	[0.048,0.056]	[0.136,0.153]	[0.308,0.347]	[0.308,0.347]
A_2	[0.039,0.058]	[0.034,0.045]	[0.061,0.092]	[0.099,0.149]	[0.099,0.149]
	[0.012,0.024]	[0.028,0.042]	[0.019,0.038]	[0.029,0.058]	[0.029,0.058]
	[0.049,0.065]	[0.036,0.048]	[0.102,0.119]	[0.308,0.347]	[0.308,0.347]
A_3	[0.078,0.097]	[0.045,0.056]	[0.092,0.122]	[0.099,0.149]	[0.099,0.149]
	[0.073,0.097]	[0.042,0.056]	[0.077,0.115]	[0.029,0.058]	[0.029,0.058]

Step 4. Determine the positive ideal PIS_j, negative ideal NIS_j, and average ideal AS_j in the following way. Table 9 showed the result for PIS_j, NIS_j and AS_j respectively.
Calculate PIS_j, NIS_j, AS_j for C_1 by using equations (20), (21), and (22) respectively.

$$(PIS_{b_j}) = \left[\max(0.049, 0.086, 0.049) = 0.086, \max(0.065, 0.097, 0.065) = 0.097\right]$$

$$(PIS_{I_j}) = \left[\min(0.078, 0.039, 0.078) = 0.039, \min(0.097, 0.058, 0.097) = 0.058\right]$$

$$(PIS_{s_j}) = \left[\min(0.073, 0.012, 0.073) = 0.012, \min(0.097, 0.024, 0.097) = 0.024\right]$$

$$(NIS_{b_j}) = \left[\min(0.049, 0.086, 0.049) = 0.049, \min(0.065, 0.097, 0.065) = 0.065\right]$$

$$(NIS_{I_j}) = \left[\max(0.078, 0.039, 0.078) = 0.078, \max(0.097, 0.058, 0.097) = 0.097\right]$$

$$(NIS_{s_j}) = \left[\max(0.073, 0.012, 0.073) = 0.073, \max(0.097, 0.024, 0.097) = 0.097\right]$$

$$(AS_{b_j}) = \left[avg(0.049, 0.086, 0.049) = 0.061, avg(0.065, 0.097, 0.065) = 0.075\right]$$

$$(AS_{I_j}) = \left[avg(0.078, 0.039, 0.078) = 0.065, avg(0.097, 0.058, 0.097) = 0.084\right]$$

$$(AS_{s_j}) = \left[avg(0.073, 0.012, 0.073) = 0.053, avg(0.097, 0.024, 0.097) = 0.073\right]$$

Table 9
The positive ideal (PIS_j), negative ideal (NIS_j), and average ideal (AS_j)

	C_1	C_2	C_3	C_4	C_5
	[0.086,0.097]	[0.048,0.056]	[0.136,0.153]	[0.308,0.347]	[0.308,0.347]
PIS_j	[0.039,0.058]	[0.034,0.045]	[0.061,0.092]	[0.099,0.149]	[0.099,0.149]
	[0.012,0.024]	[0.028,0.042]	[0.019,0.038]	[0.029,0.058]	[0.029,0.058]
	[0.049,0.065]	[0.036,0.048]	[0.077,0.102]	[0.077,0.154]	[0.077,0.154]
NIS_j	[0.078,0.097]	[0.045,0.056]	[0.122,0.153]	[0.297,0.347]	[0.297,0.347]
	[0.073,0.097]	[0.042,0.056]	[0.115,0.153]	[0.289,0.347]	[0.289,0.347]
	[0.061,0.075]	[0.040,0.051]	[0.105,0.125]	[0.231,0.283]	[0.231,0.283]
AS_j	[0.065,0.084]	[0.041,0.052]	[0.092,0.122]	[0.165,0.215]	[0.165,0.215]
	[0.053,0.073]	[0.037,0.051]	[0.070,0.102]	[0.116,0.154]	[0.116,0.154]

Step 5. For each alternative determine the distance from the positive ideal $d(PIS_j)$ and negative ideal $d(NIS_j)$ the solution should be defined. Also, the positive $d(AS_j^+)$ and negative $d(AS_j^-)$ distance from the average solutions should be determined.
In this step, we need to define the distance measure for IVPNS of $dE(S_j)_i$ and $dH(S_j)_i$, represent the Euclidian and Hamming distances.

Calculation the $dE(S_j)_i$ and $dH(S_j)_i$ by using equation (11) and (12)

$$dE(PIS_j)=\frac{1}{6}\begin{bmatrix}|0.086-0.049|^2+|0.097-0.065|^2+|0.039-0.078|^2+|0.058-0.097|^2\\+|0.012-0.073|^2+|0.024-0.097|^2+\ldots+C_5S_1\end{bmatrix}=0.115$$

$$dH(PIS_j)=\frac{1}{6}\begin{bmatrix}|0.086-0.049|+|0.097-0.065|+|0.039-0.078|+|0.058-0.097|\\+|0.012-0.073|+|0.024-0.097|+\ldots+C_5S_1\end{bmatrix}=0.589$$

Calculation the $dE(PIS_j)_i$ and $dH(PIS_j)_i$ by using equation (13) and (14) as follows:

$$dE(NIS_j)=\frac{1}{6}\begin{bmatrix}|0.049-0.049|^2+|0.065-0.065|^2+|0.078-0.078|^2+|0.058-0.097|^2\\+|0.073-0.073|^2+|0.097-0.097|^2+\ldots+C_5S_1\end{bmatrix}=0.000$$

$$dH(NIS_j)=\frac{1}{6}\begin{bmatrix}|0.049-0.049|+|0.065-0.065|+|0.078-0.078|+|0.058-0.097|\\+|0.073-0.073|+|0.097-0.097|+\ldots+C_5S_1\end{bmatrix}=0.000$$

The positive distance from the average solutions $dE^+(AS_j)_i$ and $dH^+(AS_j)_i$, the Euclidian and Hamming distances are calculated as follows using equations (15) and (16):

$$dE^+(AS_j)=\frac{1}{6}\begin{bmatrix}0|0.061-0.049|^2+0|0.075-0.065|^2+1|0.065-0.078|^2+1|0.084-0.097|^2\\+1|0.053-0.073|^2+1|0.073-0.097|^2+\ldots+C_5S_1\end{bmatrix}=0.035$$

$$dH^+(AS_j)=\frac{1}{6}\begin{bmatrix}0|0.061-0.049|+0|0.075-0.065|+1|0.065-0.078|+1|0.084-0.097|\\+1|0.053-0.073|+1|0.073-0.097|+\ldots+C_5S_1\end{bmatrix}=0.0251$$

The negative distance from the average solutions $dE^-(AS_j)_i$ and $dH^-(AS_j)_i$, the Euclidian and Hamming distances are calculated as follows using equations (17) and (18):

$$dE^+(AS_j)=\frac{1}{6}\begin{bmatrix}1|0.061-0.049|^2+1|0.075-0.065|^2+0|0.065-0.078|^2+0|0.084-0.097|^2\\+0|0.053-0.073|^2+0|0.073-0.097|^2+\ldots+C_5S_1\end{bmatrix}=0.014$$

$$dH^-(AS_j)=\frac{1}{6}\begin{bmatrix}1|0.061-0.049|+1|0.075-0.065|+0|0.065-0.078|+0|0.084-0.097|\\+0|0.053-0.073|+0|0.073-0.097|+\ldots+C_5S_1\end{bmatrix}=0.108$$

In the Table 10 below, represent the result of Euclidian and Hamming distances distance from the positive, negative, and average solution for alternatives.

Table 10
The Euclidian and Hamming distances distance from the positive, negative, and average solution

Alt.	$dE(PIS_j)_i$	$dH(PIS_j)_i$	$dE(NIS_j)_i$	$dH(NIS_j)_i$
A_1	0.115	0.589	0.000	0.000
A_2	0.000	0.000	0.115	0.589
A_3	0.005	0.102	0.108	0.487
Alt.	$dE^+(AS_j)_i$	$dH^+(AS_j)_i$	$dE^-(AS_j)_i$	$dH^-(AS_j)_i$
A_1	0.035	0.251	0.014	0.108
A_2	0.004	0.067	0.011	0.163
A_3	0.004	0.065	0.009	0.112

The calculation of σ represent the correction coefficient obtained by using equation (24):

$$\sigma(PIS) = \max(0.115, 0.000, 0.005) - \min(0.115, 0.000, 0.005) = 0.115$$
$$\sigma(NIS) = \max(0.000, 0.115, 0.108) - \min(0.000, 0.115, 0.108) = 0.115$$
$$\sigma(AS^+) = \max(0.035, 0.004, 0.004) - \min(0.035, 0.004, 0.004) = 0.032$$
$$\sigma(AS^+) = \max(0.014, 0.011, 0.009) - \min(0.014, 0.011, 0.009) = 0.005$$

The distance from the positive ideal $d(PIS_j)$ and negative ideal $d(NIS_j)$ the solution should be defined. Also, the positive $d(AS_j^+)$ and negative $d(AS_j^-)$ distance from the average solution presented in Table 11. The calculation of $d(PIS_j)$, $d(NIS_j)$, $d(AS_j^+)$ and $d(AS_j^-)$ calculate using equation 23 as follows:

$$d(PIS_j) = 0.115 + 0.115 \times 0.115 \times 0.589 = 0.123$$
$$d(PIS_j) = 0.000 + 0.115 \times 0.000 \times 0.000 = 0.000$$
$$d(AS^+{}_j) = 0.035 + 0.032 \times 0.035 \times 0.251 = 0.036$$
$$d(AS^-{}_j) = 0.014 + 0.005 \times 0.014 \times 0.108 = 0.014$$

Table 11
The positive Ideal $d(PIS_j)$, negative Ideal $d(NIS_j)$ and the positive $d(AS_j^+)$ and negative distance from the average solution

	$d(PIS_j)_i$	$d(NIS_j)_i$	$d(AS^+{}_j)_i$	$d(AS^-{}_j)_i$
A_1	0.123	0.000	0.036	0.014
A_2	0.000	0.123	0.004	0.011
A_3	0.005	0.114	0.004	0.009

Step 6. Rank the alternatives in ascending order based on the comprehensive distance showed in Table 12. Calculate the rank of the alternative by using equation (25) as follows:

$$S_1 = \frac{0.123 - 0.000 - 0.036 + 0.014}{4} = 0.025$$

Table 12
The result gained by using the COBRA-IVPNS method

Alt.	dC	RANK
A_1	0.025	3
A_2	-0.029	1
A_3	-0.026	2

The results indicate that strategy S_2, focused on social e-commerce adoption model, receives the highest preference for the application under current conditions, whereas strategy S_1, which involved E-customization and personalization, received while the least favourable response.

4.3 Likert scale of IVPNS

The illustrative example of IT supplier selection used in this study is adapted from Shohaimy *et al.,* [53]. It considers nine sub-criteria and evaluates four suppliers. The sub-criteria are based on

Shohaimay *et al.,* [53], and include supply performance (K_1), relevant experience (K_2), product quality K_3), technical support (K_4), warranty (K_5), product price (K_6), delivery time (K_7), location of the firm (K_8), and specification compliant (K_9), respectively. These criteria are divided into benefit criteria (BC) and cost criteria (C). The IT supplier selection process involves one decision maker and four alternative decisions. Four suppliers are being selected according to nine set criteria: K_1, K_2, K_3, K_4, and K_5, are classified as benefit criteria, and K_6, K_7, K_8, and K_9 are categorized as cost criteria, illustrated in Figure 3 below.

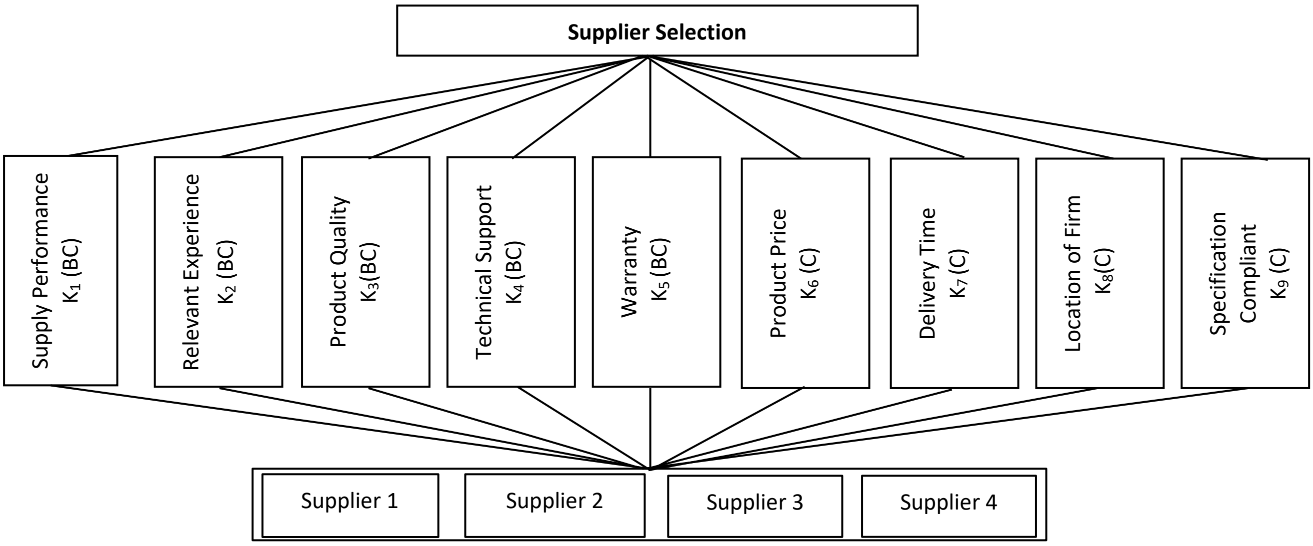

Fig. 3. IT Supplier Selection

The decision-making for IT supplier selection involved only one decision-maker maker and his rating using IVPNS $x=[(b_A^L,b_A^U),(I_A^L,I_A^U),(s_A^L,s_A^U)$ is presented in Table 13 below.

Table 13
Decision-Makers Rating of the supplier selection

	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9
	[0.60,0.70]	[0.35,0.40]	[0.75,0.85]	[0.35,0.40]	[0.60,0.70]	[0.50,0.60]	[0.40,0.50]	[0.40,0.50]	[0.50,0.60]
S_1	[0.20,0.30]	[0.45,0.50]	[0.10,0.20]	[0.45,0.50]	[0.20,0.30]	[0.30,0.40]	[0.30,0.40]	[0.30,0.40]	[0.30,0.40]
	[0.20,0.25]	[0.50,0.60]	[0.10,0.15]	[0.50,0.60]	[0.20,0.25]	[0.30,0.40]	[0.40,0.45]	[0.40,0.45]	[0.30,0.40]
	[0.40,0.50]	[0.50,0.60]	[0.60,0.70]	[0.40,0.50]	[0.75,0.85]	[0.50,0.60]	[0.60,0.70]	[0.35,0.40]	[0.35,0.40]
S_2	[0.30,0.40]	[0.30,0.40]	[0.20,0.30]	[0.30,0.40]	[0.10,0.20]	[0.30,0.40]	[0.20,0.30]	[0.45,0.50]	[0.45,0.50]
	[0.40,0.45]	[0.30,0.40]	[0.20,0.25]	[0.40,0.45]	[0.10,0.15]	[0.30,0.40]	[0.20,0.25]	[0.50,0.60]	[0.50,0.60]
	[0.50,0.60]	[0.35,0.40]	[0.75,0.85]	[0.50,0.60]	[0.75,0.85]	[0.60,0.70]	[0.40,0.50]	[0.50,0.60]	[0.40,0.50]
S_3	[0.30,0.40]	[0.45,0.50]	[0.10,0.20]	[0.30,0.40]	[0.10,0.20]	[0.20,0.30]	[0.30,0.40]	[0.30,0.40]	[0.30,0.40]
	[0.30,0.40]	[0.50,0.60]	[0.10,0.15]	[0.30,0.40]	[0.10,0.15]	[0.20,0.25]	[0.40,0.45]	[0.30,0.40]	[0.40,0.45]
	[0.50,0.60]	[0.20,0.30]	[0.40,0.50]	[0.60,0.70]	[0.60,0.70]	[0.40,0.50]	[0.75,0.85]	[0.40,0.50]	[0.50,0.60]
S_4	[0.30,0.40]	[0.50,0.55]	[0.30,0.40]	[0.20,0.30]	[0.20,0.30]	[0.30,0.40]	[0.10,0.20]	[0.30,0.40]	[0.30,0.40]
	[0.30,0.40]	[0.60,0.65]	[0.40,0.45]	[0.20,0.25]	[0.20,0.25]	[0.40,0.45]	[0.10,0.15]	[0.40,0.45]	[0.30,0.40]

Table 14
The Criteria Weight of Supplier Selection

	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9
w_j	0.16	0.04	0.18	0.08	0.13	0.05	0.17	0.02	0.17

As Table 14 shows, the weight of nine criteria where the highest value is Product quality (K_3). To attain the ultimate ranking order of the IT supplier's selection, the COBRA-IVPNS method is employed using Step 1 until Step 6.

Step 1. Forming the decision makers rating of the supplier selection in Table 13 to the decision matrix A.

Step 2. Form the normalized IVPNS in decision matrix A using Equation (9), for Benefit Criteria and Equation (10) for Cost Criteria.

The calculation for K_1 until K_5 using equation (9) for K_1S_1 calculation as follows:

$$b_{K_1S_1} = \max(0.6000, 0.4000, 0.5000, 0.5000, 0.7000, 0.5000, 0.6000, 0.6000) = 0.7000$$

$$\frac{0.6000}{0.7000} = 0.8571, \frac{0.7000}{0.7000} = 1.0000$$

$$I_{K_1S_1} = \max(0.2000, 0.3000, 0.3000, 0.3000, 0.3000, 0.4000, 0.4000, 0.4000) = 0.4000$$

$$\frac{0.2000}{0.4000} = 0.5000, \frac{0.2000}{0.4000} = 0.7500$$

$$s_{K_1S_1} = \max(0.2000, 0.4000, 0.3000, 0.3000, 0.2500, 0.4500, 0.4000, 0.4000) = 0.4500$$

$$\frac{0.2000}{0.4500} = 0.4444, \frac{0.2500}{0.4500} = 0.5556$$

In Table 15 showed the normalized IVPNS-COBRA for Benefit Criteria.

Table 15
The Normalized IVPNS-COBRA for Benefit (K_1- K_5) and Cost Criteria (K_6- K_9)

	K_1	K_2	K_3	K_4	K_5
1	[0.8571,1.0000]	[0.5833,0.6667]	[0.8824,1.0000]	[0.5000,0.5714]	[0.7059,0.8235]
	[0.5000,0.7500]	[0.8182,0.9091]	[0.2500,0.5000]	[0.9000,1.0000]	[0.6667,1.0000]
	[0.4444,0.5556]	[0.7692,0.9231]	[0.2222,0.3333]	[0.8333,1.0000]	[0.8000,1.0000]
2	[0.5714,0.7143]	[0.8333,1.0000]	[0.7059,0.8235]	[0.5714,0.7143]	[0.8824,1.0000]
	[0.7500,1.0000]	[0.5455,0.7273]	[0.5000,0.7500]	[0.6000,0.8000]	[0.3333,0.6667]
	[0.8889,1.0000]	[0.4615,0.6154]	[0.4444,0.5556]	[0.6667,0.7500]	[0.4000,0.6000]
3	[0.7143,0.8571]	[0.5833,0.6667]	[0.8824,1.0000]	[0.7143,0.8571]	[0.8824,1.0000]
	[0.7500,1.0000]	[0.8182,0.9091]	[0.2500,0.5000]	[0.6000,0.8000]	[0.3333,0.6667]
	[0.6667,0.8889]	[0.7692,0.9231]	[0.2222,0.3333]	[0.5000,0.6667]	[0.4000,0.6000]
4	[0.7143,0.8571]	[0.3333,0.5000]	[0.4706,0.5882]	[0.8571,1.0000]	[0.7059,0.8235]
	[0.7500,1.0000]	[0.9091,1.0000]	[0.7500,1.0000]	[0.4000,0.6000]	[0.6667,1.0000]
	[0.6667,0.8889]	[0.9231,1.0000]	[0.8889,1.0000]	[0.3333,0.4167]	[0.8000,1.0000]

The calculation for K_6 until K_9 using equation (10) for K_6S_1 as follows:

$$b_{K_6S_1} = \min(0.5000, 0.6000, 0.5000, 0.6000, 0.6000, 0.7000, 0.4000, 0.5000) = 0.4000$$

$$\frac{0.4000}{0.6000} = 0.6667, \frac{0.4000}{0.5000} = 0.8000$$

$$I_{K_6S_1} = \min(0.3000, 0.4000, 0.3000, 0.4000, 0.2000, 0.3000, 0.3000, 0.4000) = 0.2000$$

$$\frac{0.2000}{0.4000} = 0.5000, \frac{0.2000}{0.3000} = 0.6667$$

$$s_{K_6S_1} = \max(0.3000, 0.4000, 0.3000, 0.4000, 0.2000, 0.2500, 0.4000, 0.4500) = 0.2000$$

$$\frac{0.2000}{0.4000} = 0.5000, \frac{0.2000}{0.3000} = 0.6667$$

Table 16 showed the normalized IVPNS-COBRA for Cost Criteria.

Table 16
The Normalized IVPNS-COBRA for Cost Criteria (K_6- K_9)

	K_6	K_7	K_8	K_9
	[0.6667,0.8000]	[0.8000,1.0000]	[0.7000,0.8750]	[0.5833,0.7000]
S_1	[0.5000,0.6667]	[0.2500,0.3333]	[0.7500,1.0000]	[0.7500,1.0000]
	[0.5000,0.6667]	[0.2222,0.2500]	[0.6667,0.7500]	[0.7500,1.0000]
	[0.6667,0.8000]	[0.5714,0.6667]	[0.8750,1.0000]	[0.8750,1.0000]
S_2	[0.5000,0.6667]	[0.3333,0.5000]	[0.6000,0.6667]	[0.6000,0.6667]
	[0.5000,0.6667]	[0.4000,0.5000]	[0.5000,0.6000]	[0.5000,0.6000]
	[0.5714,0.6667]	[0.8000,1.0000]	[0.5833,0.7000]	[0.7000,0.8750]
S_3	[0.6667,1.0000]	[0.2500,0.3333]	[0.7500,1.0000]	[0.7500,1.0000]
	[0.8000,1.0000]	[0.2222,0.2500]	[0.7500,1.0000]	[0.6667,0.7500]
	[0.8000,1.0000]	[0.4706,0.5333]	[0.7000,0.8750]	[0.5833,0.7000]
S_4	[0.5000,0.6667]	[0.5000,1.0000]	[0.7500,1.0000]	0.7500,1.0000]
	[0.4444,0.5000]	[0.6667,1.0000]	[0.6667,0.7500]	0.7500,1.0000]

Step 3. Form the weight of normalized IVPNS in step 2 by using Equation (19).

Calculation for K_1S_1, in this step we multiply the normalized IVPNS with the criteria given in Table 17.

Table 17
The Weight Normalize of COBRA-IVPNS

	K_1	K_2	K_3	K_4	K_5
	[0.1371,0.1600]	[0.0233,0.0267]	[0.1588,0.1800]	[0.0400,0.0457]	[0.0918,0.1071]
S_1	[0.0800,0.1200]	[0.0327,0.0364]	[0.0450,0.0900]	[0.0720,0.0800]	[0.0867,0.1300]
	[0.0711,0.0889]	[0.0308,0.0369]	[0.0400,0.0600]	[0.0667,0.0800]	[0.1040,0.1300]
	[0.0914,0.1143]	[0.0333,0.0400]	[0.1271,0.1482]	[0.0457,0.0571]	[0.1147,0.1300]
S_2	[0.1200,0.1600]	[0.0218,0.0291]	[0.0900,0.1350]	[0.0480,0.0640]	[0.0433,0.0867]
	[0.1422,0.1600]	[0.0185,0.0246]	[0.0800,0.1000]	[0.0533,0.0600]	[0.0520,0.0780]
	[0.1143,0.1371]	[0.0233,0.0267]	[0.1588,0.1800]	[0.0571,0.0686]	[0.1147,0.1300]
S_3	[0.1200,0.1600]	[0.0327,0.0364]	[0.0450,0.0900]	[0.0480,0.0640]	[0.0433,0.0867]
	[0.1067,0.1422]	[0.0308,0.0369]	[0.0400,0.0600]	[0.0400,0.0533]	[0.0520,0.0780]
	[0.1143,0.1371]	[0.0133,0.0200]	[0.0847,0.1059]	[0.0686,0.0800]	[0.0918,0.1071]
S_4	[0.1200,0.1600]	[0.0364,0.0400]	[0.1350,0.1800]	[0.0320,0.0480]	[0.0867,0.1300]
	[0.1067,0.1422]	[0.0369,0.0400]	[0.1600,0.1800]	[0.0267,0.0333]	[0.1040,0.1300]

	K_6	K_7	K_8	K_9
	[0.0333,0.0400]	[0.1360,0.1700]	[0.0140,0.0175]	[0.0992,0.1190]
S_1	[0.0250,0.0333]	[0.0425,0.0567]	[0.0150,0.0200]	[0.1275,0.1700]
	[0.0250,0.0333]	[0.0378,0.0425]	[0.0133,0.0150]	[0.1275,0.1700]
	[0.0333,0.0267]	[0.0971,0.1133]	[0.0175,0.0200]	[0.1488,0.1700]
S_2	[0.0167,0.0222]	[0.0567,0.0850]	[0.0120,0.0133]	[0.1020,0.1133]
	[0.0167,0.0222]	[0.0680,0.0850]	[0.0100,0.0120]	[0.0850,0.1020]
	[0.0286,0.0222]	[0.1360,0.1700]	[0.0117,0.0140]	[0.1190,0.1488]
S_3	[0.0222,0.0333]	[0.0425,0.0567]	[0.0150,0.0200]	[0.1275,0.1700]
	[0.0267,0.0333]	[0.0378,0.0425]	[0.0150,0.0200]	[0.1133,0.1275]
	[0.0400,0.0286]	[0.0800,0.0907]	[0.0140,0.0175]	[0.0992,0.1190]
S_4	[0.0143,0.0190]	[0.0850,0.1700]	[0.0150,0.0200]	[0.1275,0.1700]
	[0.0127,0.0143]	[0.1133,0.1700]	[0.0133,0.0150]	[0.1275,0.1700]

Step 4. Determine the positive ideal (PIS_j), negative ideal (NIS_j), and average ideal (AS_j) in the following way. The result for (PIS_j), (NIS_j) and (AS_j) presented in Table 18.

Calculate PIS_j, NIS_j, AS_j for K_1 by using equations (20), (21), and (22) respectively. The calculation for K_1 is as follows:

$$(PIS_{b,k_1}) = [\max(0.1371, 0.0914, 0.1143, 0.1143) = 0.1371, \max(0.1600, 0.1143, 0.1371, 0.1371)] = 0.1600$$

$$(PIS_{I,k_1}) = [\min(0.0800, 0.1200, 0.1200, 0.1200) = 0.1600, \min(0.1200, 0.1600, 0.1600, 0.1600)] = 0.1200$$

$$(PIS_{s,k_1}) = [\min(0.0711, 0.1422, 0.1067, 0.1067) = 0.0711, \min(0.0889, 0.1600, 0.1422, 0.1422)] = 0.0899$$

$$(NIS_{b,k_1}) = [\min(0.1371, 0.0914, 0.1143, 0.1143) = 0.0914, \min(0.1600, 0.1143, 0.1371, 0.1371)] = 0.1143$$

$$(NIS_{I,k_1}) = [\max(0.0800, 0.1200, 0.1200, 0.1200) = 0.1200, \max(0.1200, 0.1600, 0.1600, 0.1600)] = 0.1600$$

$$(NIS_{s,k_1}) = [\max(0.0711, 0.1422, 0.1067, 0.1067) = 0.1422, \min(0.0889, 0.1600, 0.1422, 0.1422)] = 0.1600$$

$$(AS_{b,k_1}) = [avg(0.1371, 0.0914, 0.1143, 0.1143) = 0.1143, avg(0.1600, 0.1143, 0.1371, 0.1371)] = 0.1371$$

$$(AS_{I,k_1}) = [avg(0.0800, 0.1200, 0.1200, 0.1200) = 0.1100, avg(0.1200, 0.1600, 0.1600, 0.1600)] = 0.1500$$

$$(AS_{s,k_1}) = [avg(0.0711, 0.1422, 0.1067, 0.1067) = 0.1067, avg(0.0889, 0.1600, 0.1422, 0.1422)] = 0.1333$$

Table 18
The positive ideal (PIS_j), negative ideal (NIS_j), and average ideal (AS_j) of COBRA - IVPNS

	K_1	K_2	K_3	K_4	K_5
(PIS_j)	[0.1371,0.1600]	[0.0333,0.0400]	[0.1588,0.1800]	[0.0686,0.0800]	[0.1147,0.1300]
	[0.0800,0.1200]	[0.0218,0.0291]	[0.0450,0.0900]	[0.0320,0.0480]	[0.0433,0.0867]
	[0.0711,0.0889]	[0.0185,0.0246]	[0.0400,0.0600]	[0.0267,0.0333]	[0.0520,0.0780]
(NIS_j)	[0.0914,0.1143]	[0.0133,0.0200]	[0.0847,0.1059]	[0.0400,0.0457]	[0.0918,0.1071]
	[0.1200,0.1600]	[0.0364,0.0400]	[0.1350,0.1800]	[0.0720,0.0800]	[0.0867,0.1300]
	[0.1422,0.1600]	[0.0369,0.0400]	[0.1600,0.1800]	[0.0667,0.0800]	[0.1040,0.1300]
(AS_j)	[0.1143,0.1371]	[0.0233,0.0283]	[0.1324,0.1535]	[0.0529,0.0629]	[0.1032,0.1185]
	[0.1100,0.1500]	[0.0309,0.0355]	[0.0788,0.1238]	[0.0500,0.0640]	[0.0650,0.1083]
	[0.1067,0.1333]	[0.0292,0.0346]	[0.0800,0.1000]	[0.0467,0.0567]	[0.0780,0.1040]
	K_6	K_7	K_8	K_9	
(PIS_j)	[0.0400,0.0400]	[0.1360,0.1700]	[0.0175,0.0200]	[0.1488,0.1700]	
	[0.0143,0.0190]	[0.0425,0.0567]	[0.0120,0.0133]	[0.1020,0.1133]	
	[0.0127,0.0143]	[0.0378,0.0425]	[0.0100,0.0120]	[0.0850,0.1020]	
(NIS_j)	[0.0286,0.0222]	[0.0800,0.0907]	[0.0117,0.0140]	[0.0992,0.1190]	
	[0.0250,0.0333]	[0.0850,0.1700]	[0.0150,0.0200]	[0.1275,0.1700]	
	[0.0267,0.0333]	[0.1133,0.1700]	[0.0150,0.0200]	[0.1275,0.1700]	
(AS_j)	[0.0338,0.0294]	[0.1123,0.1360]	[0.0143,0.0173]	[0.1165,0.1392]	
	[0.0195,0.0270]	[0.0567,0.0921]	[0.0143,0.0183]	[0.1211,0.1558]	
	[0.0203,0.0258]	[0.0642,0.0850]	[0.0129,0.0155]	[0.1133,0.1424]	

Step 5. For each alternative determines the distance from the positive ideal $d(PIS_j)$ and negative ideal $d(NIS_j)$. Also, the positive $d(AS_j^+)$ and negative $d(AS_j^-)$ distance from the average solutions should be determined.

In this step, we need to define the distance measure for IVPNS of $dE(S_j)_i$ and $dH(S_j)_i$, represent the Euclidian and Hamming distances.

The $dE(PIS_j)_i$ and $dH(PIS_j)_i$ are calculated by using equation (11) and (12) as follows:

$$dE(PIS_{jS_1})=\frac{1}{6}\begin{bmatrix}|0.1371-0.1371|^2+|0.1600-0.1600|^2+|0.0800-0.0800|^2\\+|0.1200-0.1200|^2+|0.0711-0..0711|^2+|0.0899-0.0899|^2+\ldots+K_9s_1\end{bmatrix}=0.0059$$

$$dE(PIS_{jS_1})=\frac{1}{6}\begin{bmatrix}|0.1371-0.1371|+|0.1600-0.1600|+|0.0800-0.0800|\\+|0.1200-0.1200|+|0.0711-0..0711|+|0.0899-0.0899|+\ldots+K_9s_1\end{bmatrix}=0.1504$$

Calculate the $dE(NIS_j)_i$ and $dH(NIS_j)_i$ by using equation (13) and (14) as follows:

$$dE(NIS_{jS_1})=\frac{1}{6}\begin{bmatrix}|0.0914-0.1371|^2+|0.1143-0.1600|^2+|0.1200-0.0800|^2\\+|0.1600-0.1200|^2+|0.1422-0..0711|^2+|0.1600-0.0899|^2+\ldots+K_9s_1\end{bmatrix}=0.0200$$

$$dE(NIS_{jS_1})=\frac{1}{6}\begin{bmatrix}|0.0914-0.1371|+|0.1143-0.1600|+|0.1200-0.0800|\\+|0.1600-0.1200|+|0.1422-0..0711|+|0.1600-0.0899|+\ldots+K_9s_1\end{bmatrix}=0.2410$$

The Euclidian and Hamming distances for positive distance from the average solutions $dE^+(AS_j)_i$ and $dH^+(AS_j)_i$, are calculated as follows using equations (15) and (16):

$$dE^+(NIS_{jS_1})=\frac{1}{6}\begin{bmatrix}1|0.1143-0.1371|^2+1|0.1371-0.1600|^2+0|0.1100-0.0800|^2\\+0|0.1500-0.1200|^2+0|0.1067-0..0711|^2+0|0.1333-0.0899|^2+\ldots+K_9s_1\end{bmatrix}=0.0016$$

$$dH^+(NIS_{jS_1})=\frac{1}{6}\begin{bmatrix}1|0.1143-0.1371|+1|0.1371-0.1600|+0|0.1100-0.0800|\\+0|0.1500-0.1200|+0|0.1067-0..0711|+0|0.1333-0.0899|+\ldots+K_9s_1\end{bmatrix}=0.0733$$

The Euclidian and Hamming distances for negative distance from the average solutions $dE^-(AS_j)_i$ and $dH^-(AS_j)_i$, are calculated as follows using equations (17) and (18):

$$dE^-(NIS_{jS_1})=\frac{1}{6}\begin{bmatrix}0|0.1143-0.1371|^2+0|0.1371-0.1600|^2+1|0.1100-0.0800|^2\\+1|0.1500-0.1200|^2+1|0.1067-0..0711|^2+1|0.1333-0.0899|^2+\ldots+K_9s_1\end{bmatrix}=0.0027$$

$$dH^-(NIS_{jS_1})=\frac{1}{6}\begin{bmatrix}0|0.1143-0.1371|+0|0.1371-0.1600|+1|0.1100-0.0800|\\+1|0.1500-0.1200|+1|0.1067-0..0711|+1|0.1333-0.0899|+\ldots+K_9s_1\end{bmatrix}=0.0832$$

In Table 19 showed the Euclidian and Hamming distances for $(PIS_j),(NIS_j),(AS_j)^+$, and $(AS_j)^-$.

Table 19

The Euclidian and Hamming distances for $(PIS_j),(NIS_j),(AS_j)^+$, and $(AS_j)^-$

	$dE(PIS_j)_i$	$dH(PIS_j)_i$	$dE(NIS_j)_i$	$dH(NIS_j)_i$
S_1	0.0059	0.1504	0.0200	0.2410
S_2	0.0064	0.1544	0.0113	0.2370
S_3	0.0031	0.1125	0.0202	0.2789
S_4	0.0230	0.3233	0.0020	0.0681
	$dE^+(AS_j)_i$	$dH^+(AS_j)_i$	$dE^-(AS_j)_i$	$dH^-(AS_j)_i$
S_1	0.0016	0.0733	0.0027	0.0832
S_2	0.0008	0.0387	0.0016	0.0683
S_3	0.0007	0.0406	0.0020	0.0680
S_4	0.0067	0.1274	0.0018	0.0605

The calculation of σ represent the correction coefficient obtained by using equation (24):

$$\sigma(PIS) = \max(0.0059, 0.0064, 0.0031, 0.0230) - \min(0.0059, 0.0064, 0.0031, 0.0230) = 0.0199$$
$$\sigma(NIS) = \max(0.0200, 0.0113, 0.0202, 0.0020) - \min(0.0200, 0.0113, 0.0202, 0.0020) = 0.0182$$
$$\sigma(AS^{+}) = \max(0.0016, 0.0008, 0.0007, 0.0067) - \min(0.0016, 0.0008, 0.0007, 0.0067) = 0.060$$
$$\sigma(AS^{-}) = \max(0.0027, 0.0016, 0.020, 0.0018) - \min(0.0027, 0.0016, 0.020, 0.0018) = 0.0011$$

The distance from the positive ideal $d(PIS_j)$ and negative ideal $d(NIS_j)$ the solution should be defined. Also, the positive $d(AS_j^{+})$ and negative $d(AS_j^{-})$ distance from the average solution presented in Table 20. The calculation of $d(PIS_j)$, $d(NIS_j)$, $d(AS_j^{+})$ and $d(AS_j^{-})$ calculate using equation (25) as follows:

$$d(PIS_{S_1}) = 0.0059 + 0.0199 \times 0.0059 \times 0.1504 = 0.0060$$
$$d(NIS_{S_1}) = 0.0200 + 0.0182 \times 0.0200 \times 0.2410 = 0.0201$$
$$d^{+}(AS_{S_1}) = 0.0016 + 0.0008 \times 0.0060 \times 0.0733 = 0.0016$$
$$d^{-}(AS_{S_1}) = 0.0027 + 0.0005 \times 0.0027 \times 0.0832 = 0.0027$$

Table 20
The Positive Ideal $d(PIS_j)$, Negative Ideal $d(NIS_j)$ and The Positive $d(AS_j^{+})$ and Negative Distance from The Average Solution

	$d(PIS_j)_i$	$d(NIS_j)_i$	$d(AS^{+}{}_j)_i$	$d(AS^{-}{}_j)_i$
S_1	0.0060	0.0201	0.0016	0.0027
S_2	0.0064	0.0114	0.0008	0.0016
S_3	0.0031	0.0203	0.0007	0.0020
S_4	0.0231	0.0020	0.0067	0.0018

Step 6. Rank the alternatives in ascending order based on the comprehensive distance.
Calculate the rank of the alternative by using equation (25) as follows:

$$S_1 = \frac{0.0060 - 0.0201 - 0.0016 + 0.0027}{4} = -0.00327$$

Table 21
The result gained by using the COBRA – IVPNS method for IT supplier selection

Supplier	dC	RANK
S_1	-0.00327	2
S_2	-0.00106	3
S_3	-0.00398	1
S_4	0.00404	4

The findings of this study indicate in Table 21, show that the top choices for selecting suppliers within the IT department are Supplier 2 (S_3) as the most highly recommended option, followed by Supplier 3 (S_1), Supplier 1 (S_2), and Supplier 4 (S_4).

4.4 Comparison of COBRA-IVPNS with COBRA

In this section, a comparative analysis is conducted using 5 linguistic variables, applying the same criteria weight values from Popović [44]. The results are compared with those from the new IVPNS-

COBRA method and the original COBRA method using 5 linguistic variables for IVPNS. Additionally, the comparison includes the Neutrosophic Set (PNS) using the original COBRA method, based on 5 linguistic variables adopted from Kamari *et al.,* [39] as well as the results from MEREC-COBRA [44]. For the 7 linguistic variables, the case study similarly applies the same criteria weight values for both IVPNS and PNS. The comparison involves the new IVPNS-COBRA method alongside IVPNS with 7 linguistic variables, as well as PNS with 7 linguistic variables, as proposed by Ismail *et al*., [38] using the original COBRA method [44]. The comparison results and correlation coefficient for 5 and 7 Linguistic variables are presented in Tables 22-25 below.

Table 22
Ranking orders of decision result 5 Linguistic Variable using COBRA-IVPNS & Original COBRA

	Comprehensive Distance	Ranking
COBRA-IVPNS (NEW)	$A_1 = 0.0253, A_2 = -0.0291, A_3 = -0.0260$	$A_2 \succ A_3 \prec A_1$
IVPNS-COBRA	$A_1 = 0.1434, A_2 = -0.1175, A_3 = -0.0957$	$A_2 \succ A_3 \prec A_1$
PNS-COBRA	$A_1 = 0.3860, A_2 = -0.1946, A_3 = -0.3931$	$A_2 \succ A_3 \prec A_1$
MEREC-COBRA	$A_1 = 0.1411, A_2 = -0.1153, A_3 = -0.0923$	$A_2 \succ A_3 \prec A_1$

Table 23
Correlation Coefficient value Ranking order in Table 22

	COBRA-IVPNS (NEW)	IVPNS-COBRA	PNS-COBRA	MEREC-COBRA
COBRA-IVPNS (NEW)	1.0	1.0	0.95	1.0
IVPNS-COBRA	1.0	1.0	0.95	1.0
PNS-COBRA	0.95	0.95	1.0	0.95
MEREC-COBRA	1.0	0.95	1.0	1.0

Table 24
Ranking orders of decision result 7 Linguistic Variable using COBRA-IVPNS & Original COBRA

	Comprehensive Distance	Ranking
COBRA-IVPNS (NEW)	$S_1 = -0.0033, S_2 = 0.0011, S_3 = -0.0039, S_4 = 0.0040$	$S_2 \succ S_3 \prec S_1 \prec S_4$
IVPNS-COBRA	$S_1 = -0.1171, S_2 = 0.0004, S_3 = -0.0140, S_4 = 0.0069$	$S_2 \succ S_3 \prec S_1 \prec S_4$
PNS-COBRA	$S_1 = -0.0278, S_2 = 0.0531, S_3 = -0.0818, S_4 = 0.0058$	$S_2 \succ S_4 \prec S_1 \prec S_3$

Table 25
Correlation Coefficient value Ranking order in Table 24

	COBRA-IVPNS (NEW)	IVPNS-COBRA	PNS-COBRA
COBRA-IVPNS (NEW)	1.0	0.9	0.5
IVPNS-COBRA	0.9	1.0	0.8
PNS-COBRA	0.5	0.8	1.0

When comparing the decision result in Table 22 for the 5 linguistic variables, all method produces the same ranking order, consistently prioritizing A_2 as the best alternative, followed by A_3 and then A_1. In contrast, Table 24, which presents the results for the comparing of 7 linguistic variables, show a slight variation in the ranking when use PNS using original COBRA method in [44]. While the ranking order differs slightly, the lowest comprehensive distance value remain consistent, indicating S_2 is still identified as the best alternatives across all the methods. The correlation coefficients in Table 23 support this observation using 5 linguistic variables, showing a high correlation (almost 1.0) among IVPNS and PNS using new IVPNS-COBRA method and original COBRA method [44]. The correlation coefficients in Table 25 further support this observation, showing a high correlation between COBRA-

IVPNS (NEW) and IVPNS-COBRA (0.9), while the PNS-COBRA method exhibits some differences, resulting in a lower correlation (0.5) with COBRA-IVPNS). This suggests that despite minor ranking variations, the overall decision framework remains stable, particularly in identifying the top alternative.

5. Conclusion

This research presents a novel linguistic variable framework specifically designed for Interval-Valued Pythagorean Neutrosophic Sets (IVPNS), addressing the challenge of robust decision-making model in complex and uncertain environment. The research lays the groundwork by defining linguistic terms and developing corresponding IVPNS linguistic variables, which are subsequently evaluated using both 5-term and 7-term linguistic scales. The IVPNS, enhancing the model further, Euclidean and Hamming distance measures are incorporated to boost precision in multi-criteria decision-making applications.

The core objective was to formulate an advanced decision-making model that could effectively solved real-world problems, such as selecting e-commerce strategies and evaluating IT suppliers. By integrating IVPNS with the Comprehensive Distance-Based Ranking (COBRA) method, the resulting IVPNS-COBRA model establishes a structured and precise framework for ranking alternatives. COBRA's sophisticated ranking mechanism, which includes positive ideal, negative ideal, positive average, and negative average solutions, guarantees a comprehensive assessment of decision criteria while ensuring computational efficiency.

Ultimately, the outcomes of this research demonstrate that the IVPNS-COBRA model significantly enhances the decision-making process in environments characterized by uncertainty and incomplete information. Its seamless integration of linguistic variables and advanced distance measures offers decision-makers a reliable and valuable tool for optimizing strategic business decisions.

Future research should focus on refining the computational aspects of IVPNS-COBRA, exploring its applicability across additional industries, and integrating machine learning techniques to further enhance its predictive accuracy. Additionally, real-world case studies in digital transformation and supplier risk assessment should be conducted to validate and expand the practical utility of the proposed framework.

Author Contributions

Conceptualization, S.A.R., Z.M.R., F.A.-S., and N.R.; methodology, Z.M.R., F.A.-S., and N.R.; validation, S.A.R., Z.M.R., F.A.-S., and N.R.; investigation, S.A.R., Z.M.R., F.A.-S., and N.R.; writing—original draft preparation, S.A.R., Z.M.R., F.A.-S., and N.R.; writing—review and editing, S.A.R., Z.M.R., F.A.-S., and N.R.; visualization, S.A.R., Z.M.R., F.A.-S., and N.R. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

The data supporting the reported results in this study were derived from real-world applications involving e-commerce development strategies and IT supplier selection. The study utilized decision-making criteria and evaluation datasets adapted from established research, including sources such as Karabašević *et al.,*[48] and Shohaimy *et al.,*[49], while the processed decision matrices and ranking results are presented within the study, the raw datasets used for analysis are not publicly archived.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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