SCIENTIFIC OASIS

International Journal of Economic Sciences

Journal homepage: www.ijes-journal.org
eISSN: 1804-9796

Economic Evaluation of Digital Suppliers for Manufacturing SMEs Using Pythagorean Neutrosophic TOPSIS and VIKOR with a Flexible Distance Metric

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ARTICLE INFO

Article history:
Received 18 May 2025
Received in revised form 12 August 2025
Accepted 1 September 2025
Available online 24 September 2025

Keywords:
Pythagorean neutrosophic set; TOPSIS; VIKOR; MCDM; Economic evaluation; Manufacturing SMEs; Alternative ranking; Neutrosophic distance measure

ABSTRACT

Selecting digital suppliers is not only a technological issue but also a fundamental economic decision for manufacturing SMEs pursuing digital transformation. Supplier choice directly affects cost efficiency, resource allocation, and long-term competitiveness. This study develops a quantitative multi-criteria decision-making (MCDM) framework that integrates Pythagorean Neutrosophic TOPSIS (PNTOPSIS) and VIKOR (PNVIKOR), strengthened by a novel distance measure, the Flexible Indeterminacy Quantifier (FIQ). The framework explicitly addresses incomplete and uncertain evaluations that complicate procurement under financial and operational constraints. A real-world case with five digital suppliers is analyzed across criteria including system capability, vendor support, total cost, and risk of disruption. The findings highlight the supplier that delivers the highest economic value by aligning affordability with operational reliability. FIQ improves score differentiation and ranking stability, enabling SMEs to make more economically rational choices. Sensitivity analysis confirms that the model produces consistent outcomes under varying budgetary assumptions, demonstrating its robustness for strategic procurement. Overall, this research provides SMEs with an uncertainty-aware, cost-sensitive tool that reduces financial risks, enhances transparency in supplier selection, and supports sustainable economic performance within digitally transforming industrial ecosystems.

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https://doi.org/10.31181/ijes1412025187

1. Introduction

In an era of digital transformation, economic uncertainty, and global competition, the ability to make educated and data-driven decisions has become critical to an efficient and resilient system for manufacturing small and medium-sized enterprises (SMEs). One critical area where this need is particularly evident are in digital supplier selection (DSS). These decision affects not only cost efficiency but also technological adaptability, operational continuities, and competitive position. As SMEs strives to modernize through digital solutions, selecting the most appropriate digital supplier become a multidimensional decision-making problem that involve strategic trade-offs across various conflicting criteria. This decision is further complicated by incomplete information, subjective expert input, and uncertainty about future condition. To address this complexities, multi-criteria decision-making (MCDM) method have emerged as effective tools for support rational, transparent and uncertainty-aware analysis.

Amongst the most notable MCDM methods, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Vlsekriterijumska Optimizacija I KOmpromisno Resenje (VIKOR) are widely used to produce a systematic order of alternatives. These approaches have shown broad utility beyond supplier selection application, extending into areas such as investment analysis, energy planning, financial risk assessment, and policy prioritization further verifying their usefulness in a broad range of economic decision-making environments [1-2]. However, traditional approaches to TOPSIS and VIKOR implementations are usually based on using crisp data or rather simple fuzzy models, which do not allow the ambiguity, vagueness and hesitancy of actual cases to be defined comprehensively.

In order to overcome this limitation, the notion of neutrosophic sets and their extensions, in particular, Pythagorean neutrosophic set (PNS), have been brought forward. PNS allow decision-makers to express three degrees of membership namely, truth (τ), indeterminacy (ξ), and falsity (η), offering a more all-around portrayal of uncertain information [3-4]. Therefore, this study develops the Pythagorean neutrosophic TOPSIS (PNTOPSIS) and Pythagorean neutrosophic VIKOR (PNVIKOR) to boost the level of robustness and flexibility of ranking processes in uncertain and conflicting evaluations.

Nevertheless, a key issue is still looming in the choice of suitable distance measures within neutrosophic environments. The reliability of any neutrosophic MCDM approach largely depends on the accuracy of the distance metric used. Common distance measures such as extensions of Hamming, Euclidean, and Cosine metrics rely on rigid definitions that do not account for the complex interdependencies of truth, indeterminacy, and falsity membership degrees. These traditional measures often fail to recognize the fluid character of ambiguity and expert uncertainty, which may result in an unstable or inconsistent ranking in presence of high indeterminacy.

To fill this gap, this research proposes a new distance measure, namely Flexible Indeterminacy Quantifier (FIQ), that is specifically designed for the neutrosophic set framework. The proposed distance measure is adaptive, dynamic and provides for an accurate quantification of indeterminacy interactions, which increases the stability and precision of rankings in the neutrosophic MCDM approaches. Through improved interpretability and accuracy in distance-based assessments, the proposed measure significantly enhances the decision-making capabilities of PNTOPSIS and PNVIKOR, with digital supplier selection for manufacturing SMEs serving as a representative case application in economic decision-making.

The primary contributions of this study are:

i. Development of the FIQ distance measure.

ii. Introduction of PNTOPSIS and PNVIKOR methodologies.
iii. Demonstration of the applicability of both methods combined with FIQ measure in a digital supplier selection case for manufacturing SMEs and conducting comparative analysis.
iv. Comparison of the proposed framework with traditional distance measure and testing the ranking stability under different criterion weightage in sensitivity analysis.

The remainder of this paper is organized as follows. Section 2 examines the relevant literatures. Section 3 outlines the foundational preliminaries. Section 4 introduces the proposed FIQ distance measure. Section 5 shows the development of the PNTOPSIS and PNVIKOR approaches. Section 6 demonstrates the practical application of the proposed methodology through digital supplier selection case study, including comparative and sensitivity analyses. Finally, Section 7 provides the concluding remarks of the study.

2. Relevant Literature

This section critically examines the relevant literature in the context of this study.

2.1 Digital Supplier Selection

As a strategic requirement, digital transformation of SMEs in the manufacturing industry requires the reengineering of the organization in relation to the processes, structure, and business model with the use of technologies, including artificial intelligence (AI), the Internet of Things (IoT), and big data analytics [5-6]. These technologies can improve efficiency, enable more tailored offerings, and help businesses respond more quickly to change. However, many SMEs face real challenges due to limited funding, a lack of technical expertise, and outdated infrastructure. Even so, the development of Digital Supply Chains (DSCs) provides a promising solution. With their focus on real-time data sharing, interconnected systems, and predictive capabilities, DSCs offer SMEs a practical way to overcome these barriers and create more flexible and scalable operations [7-8].

As manufacturing SMEs adopt DSCs, digital supplier selection (DSS) becomes a vital aspect of supply chain strategy. Evaluation of the suppliers on the basis of their technological maturity, system integration capability, data transparency, and support to innovation and resilience now complement the traditional evaluation criteria, namely, cost, quality, and delivery [9-10]. These digital capabilities are essential not only for operational alignment but also for advancing Supply Chain Quality Management (SCQM), which seeks to improve quality, performance, and customer satisfaction through the coordinated efforts of supply chain partners [11]. For SMEs, selecting digitally capable suppliers is critical to building supply chains that can respond to dynamic market demands and technological disruptions.

To enable such a complex decision-making in the scenarios of DSS, researchers have developed a series of sophisticated decision-making models that can accommodate the needs of manufacturing SMEs. A hybrid SWARA-WASPAS model was proposed by Sharma and Joshi [9], in which suppliers were assessed on the basis of the following criteria innovativeness, responsiveness, interoperability, and cost effectiveness, which allowed to find out that the digital competence had a powerful positive impact on SCQM outcomes. Özek and Yildiz [12] applied interval type-2 fuzzy TOPSIS in their garment business, focusing on the importance of such criteria as digital production system and intelligent logistics, and managed the expert uncertainty effectively. Tavana *et al.,* [13] built on the DSS techniques and integrated fuzzy BWM, MULTIMOORA, COPRAS, and TOPSIS and maximize agreement heuristic (MAH) to support solid supplier prioritization in the digital environment. Erbay and Yildirm [14] provided a mixed model combining AHP and QFD and Mixed Integer Programming, with its effectiveness tested on a case of an automotive supplier SME in Turkey, where data analytics and sensor technologies were ranked the highest, and, on the contrary, the main areas of adoption challenges which included a lack of expertise and budgets constraints were identified. All of these shows together how DSS is not just an operational tool but also a strategic tool that is essential in enabling manufacturing SMEs undergoing digital transformation.

2.2 Neutrosophic sets and extension

Fuzzy set theory was first introduced by Zadeh [15], which provide the means to represent uncertainty by appointing membership values in the range [0,1]. This innovation provided a flexible mathematical framework to handle vague information. Zadeh also adapted classical set operations such as union, intersection, complement, and convexity to work within this new fuzzy framework. By using these set operations in fuzzy subsets, Zadeh built the foundation for later developments in uncertainty modeling. Despite its usefulness, fuzzy set theory lacked the capacity to adequately capture hesitation or ambiguity when information was incomplete or conflicting. To overcome these limitations, Atanassov [16] introduced intuitionistic fuzzy sets (IFS), which incorporated independent membership and non-membership degrees, along with a hesitation component that reflects the uncertainty between the two. This extension offered an improved way to express uncertain data. However, IFS still face challenges to handle inconsistent and incomplete information that often found in real world scenario. Yager [17] further extended the study by proposing Pythagorean fuzzy sets (PFS), where the squared sum of the membership and non-membership is less than or equal to one. This condition enables higher capacity and flexibility for fuzzy sets compared to IFS. Nonetheless, the ability of PFS was still limited in handling data that is consisting indeterminacy.

To address this gap, Smarandache [18] introduced neutrosophic sets (NS) which can better handle uncertainty. Unlike the previous framework, neutrosophic sets defined by three independent and unique functions namely the truth-membership, indeterminacy-membership, and falsity-membership. In environment that are full of ambiguity and contradictions, these functions support a better representation of uncertainty. The way neutrosophic sets work, the three components do not have a fixed way for their values to add up, so they can work in many flexible ways. However, Smarandache [19] and Wang *et al.,* [20] introduced the notion of the single-valued neutrosophic set (SVNS), where every value ranges from 0 to 1 and the combined sum is no greater than 3. Based on this theory, multiple new developments of neutrosophic sets have been suggested to help with different kinds of uncertainties. Some examples are interval neutrosophic sets proposed by Wang et al., [21], simplified neutrosophic sets outlined by Ye [22], neutrosophic soft sets explained by Maji [23], the multi-valued neutrosophic set by Wang and Li [24] and rough neutrosophic sets examined by Broumi *et al.,* [25]. Each variation focuses on different types of problems, while making the neutrosophic method more expressive [26].

One important addition to neutrosophic set theory is the Pythagorean neutrosophic set (PNS). Jansi *et al.,* [27] extended the correlation coefficient framework to PNS where the new framework is based on the Pythagorean constraint $0 \leq \left(\tau_A(x)\right)^2 + \left(\xi_A(x)\right)^2 + \ (\eta_A(x))^2 \leq 2$ for the components of truth, indeterminacy and falsity. Because of this constraint, PNS can cover more situations than the traditional neutrosophic model and is better suited for studying complex uncertain relationships. It is most useful in contexts where traditional neutrosophic sets may be too restrictive. PNS make aggregation and comparison processes more precise, because they allow for a detailed representation of uncertainty in decision-making. This makes it a valuable framework in applications where dealing with uncertainty is very important [28-29]. More complex types of information and hesitancy in decisions can now be captured with the bipolar Pythagorean neutrosophic set (BPNS) and the interval-valued Pythagorean neutrosophic set (IVPNS) which are both extensions of PNS [30-33]. All in all, PNS shows how uncertainty modeling is progressing to help deal with the many intricacies of real-world problems.

2.3 Neutrosophic distance measure

Decision-making, clustering, classification and information retrieval systems rely heavily on distance measures. Traditional measures, such as Euclidean and Hamming distances, are widely used across various studies. Nevertheless, they face difficulties in representing the imprecise and sometimes confusing real-world data. Neutrosophic distance measures solve these issues by bringing truth, indeterminacy and falsity degrees into one strong mathematical foundation. With these measures, it is easier to tell how alike or different the elements within neutrosophic sets are, helping handle uncertainty better.

Several neutrosophic distance measures have been used in previous research and are listed below:

1. Neutrosophic Hamming distance

$$d(A,B)=\sum_{i=1}^{n}(|\tau_A-\tau_B|+|\xi_A-\xi_B|+|\eta_A-\eta_B|) \tag{1}$$

The Hamming distance has been widely used in many neutrosophic set-based decision models because it is simple and efficient in measuring the absolute differences in truth, indeterminacy and falsity membership degrees [34-37].

2. Neutrosophic Euclidean distance

$$d(A,B)=\sqrt{\sum_{i=1}^{n}((\tau_A-\tau_B)^2+(\xi_A-\xi_B)^2+(\eta_A-\eta_B)^2)} \tag{2}$$

Unlike the Hamming distance, the Euclidean distance measures difference by working with the squared values of truth, indeterminacy and falsity. Continuing to adjust results in a refined distinction between neutrosophic sets, making it suitable for applications that need high precision [34], [36-38].

3. Neutrosophic Hausdorff metric

$$d(A,B)=max\{h_f(A,B),h_b(A,B)\} \tag{3}$$

Where $h_f(A,B)=\max_{i=1,2\dots,n}\left\{\min_{j=1,2,\dots,n} d(A_i,B_j)\right\}$ and $h_b(A,B)=\max_{j=1,2\dots,n}\left\{\min_{i=1,2,\dots,n} d(B_j,A_i)\right\}$. Here $d(a,b)$ represents the individual distance between elements a and b. In practice, this measure efficiently highlights the distinct structures found in neutrosophic sets during clustering [39].

4. Neutrosophic sine distance measure

$$d(A,B)=\frac{5}{3n}\sum_{i=1}^{n}\frac{\sin\left\{\frac{\pi}{6}|\tau_A-\tau_B|\right\}+\sin\left\{\frac{\pi}{6}|\xi_A-\xi_B|\right\}+\sin\left\{\frac{\pi}{6}|\eta_A-\eta_B|\right\}}{1+\sin\left\{\frac{\pi}{6}|\tau_A-\tau_B|\right\}+\sin\left\{\frac{\pi}{6}|\xi_A-\xi_B|\right\}+\sin\left\{\frac{\pi}{6}|\eta_A-\eta_B|\right\}} \tag{4}$$

A new approach to measuring distance in neutrosophic sets is the sine distance measure which uses a non-linear transformation that works better with uncertainty. This choice of function helps to highlight smaller differences that affect the total distance in a non-linear way which is helpful in complex decision-making cases [40].

5. Generalized weighted distance measure

$$d(A,B) = \left\{\sum_{i=1}^{n} w_i[|\tau_A - \tau_B|^p + |\xi_A - \xi_B|^p + |\eta_A - \eta_B|^p]\right\}^{\frac{1}{p}} \tag{5}$$

This distance measure which derived from the Minkowski formulation proposed by Ye [34], has been widely applied in neutrosophic decision-making. The measure gives a score based on weights that do not change and a fixed power parameter p. Although this approach is fundamental, it is not flexible enough to deal with different levels of uncertainty. Therefore, we introduce the FIQ distance which is based on the generalized model but brings major upgrades: adaptive weightings, self-regulating exponents and dynamic norm selection. With these features, FIQ is more responsive and robust in environments involving high uncertainty.

2.4 MCDM

Multi-criteria decision-making (MCDM) is a vital branch of decision science that provides a structured and systematic approach for evaluating complex problems involving multiple, often conflicting criteria. MCDM is a major area of decision science that helps organize and systematically solve issues where several, sometimes clashing, criteria come into play. MCDM has become necessary for making fair and clear decisions, mainly because challenges for decision-making are inevitable in strategic management, healthcare, environmental sustainability and engineering [41]. This helps decision-makers to take into account both quantitative data and qualitative judgements and rate each aspect in line with its importance. Looking at economic issues from all angles helps lead to sensible and contextual decisions [42].

A central function of MCDM is the ranking of alternatives that allows for sorting of systematic ranking of alternatives in a way that suits the main objectives. This process is essential when resources are scarce, when different groups have different interests or when the results matter a lot [43]. A high-quality ranking helps make the results more transparent, promotes reproducibility and supports sensitivity analysis. All of these factors play a key role in helping decisions to be reliable and supportable [44].

In the world of MCDM, TOPSIS and VIKOR are widely utilized in theoretical studies and practical applications. Their value lies in handling different choices, reconciling events with conflicting goals and handling uncertainty [45]. This study uses both methods together to take advantage of their strengths, aiming to produce clear and well-rounded rankings of the alternatives. The latest development is the creation of the Adaptive Utility Ranking Algorithm (AURA) as a new MCDM ranking method that incorporates a flexible distance-based approach, streamlines computations, and applies a dynamic normalization scheme capable of handling benefit, cost, and target-type criteria for improved adaptability in real-world decision-making [46].

Recently, more studies have applied neutrosophic extensions in MCDM problems. For example, Abdalla *et al.,* [47] developed interval-valued Fermatean neutrosophic super hypersoft sets to help with the uncertainties that occur in healthcare decisions. Biswas *et al.,* [48] use triangular neutrosophic numbers within the CRITIC-COPRAS framework to assess potential canteen sites, while Basuri *et al.,* [49] used new scoring functions based on neutrosophic numbers to guide school location selection. These studies demonstrate that neutrosophic models are gaining flexibility and importance for resolving various MCDM cases.

The next part reviews official literature dealing with TOPSIS and VIKOR and gives the necessary introduction for their application in this study.

2.4.1 TOPSIS

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), originally introduced by Hwang and Yoon [50], is still considered a major technique in MCDM. It stands out most because it orders options based on their closeness to an ideal solution, whilst still handling the differences between positive and negative ideal points. TOPSIS is different from pairwise methods since it easily handles complex decision situations by providing a straightforward ranking tool. Its strong performance allows it to be applied in many domains, like engineering, finance, management and operations research, where it helps in optimizing product design, facility location selection, and resource allocation.

Since decision-making situations are increasingly complex, researchers have sought to make TOPSIS more accurate by blending it with fuzzy set theory. According to recent studies, combining several methods brings better results. By fusing fuzzy Analytical Hierarchy Process (AHP), fuzzy TOPSIS, and clustering methods, Arıcan and Kara optimized the choice of chemical tankers for different cargo types, outlining a clear selection process [51]. Likewise, as detailed by He *et al.,* [52], a hybrid fuzzy AHP-TOPSIS framework was used to review music education strategies that assist in overcoming anxiety and depression in Chinese university students. Otay *et al.,* [53] combined fuzzy TOPSIS with interval-valued Pythagorean fuzzy sets and multi-expert fuzzy Best-Worst Method (BWM) to determine the most appropriate investments in sustainable energy for smart cities. Moreover, Usun *et al.,* [54] used type-2 fuzzy sets within an extended TOPSIS framework for their study and contributed to better passenger satisfaction in the airline industry by improving customer segmentation and higher service quality ratings. Additionally, Quynh [55] introduced an integral-value fuzzy TOPSIS model for banking performance evaluation with a normalization process for ranking fuzzy numbers which leads to more accurate results in financial assessments.

Recent work has improved TOPSIS further by introducing neutrosophic sets to deal with uncertain cases that have both indeterminacy and inconsistent information. Sharma *et al.,* [56] introduced interval-valued neutrosophic TOPSIS to rank hotels based on Tripadvisor aspect ratings, evidence its applicability for studying consumer preferences. Argilagos *et al.,* [57] used neutrosophic TOPSIS to arrange strategies aimed at promoting healthy nutritional habits in schools, efficiently handling unclear data in behavioral research. Recalde *et al.,* [58] showed that SVNS-TOPSIS is effective by using it to assess preventive dental education programs, thus confirming its usefulness for educational planning. Moreover, Anwar *et al.,* [59] introduced a neutrosophic TOPSIS system for evaluating sports league performances, resolving inconsistencies in player performance statistics and offering a more reliable approach to ranking the teams.

These developments highlight how TOPSIS methods are becoming more advanced in dealing with uncertainty. Fuzzy and neutrosophic extensions have increase the applicability of TOPSIS methods and made it possible to address challenges that are hard to define and for decision-makers to manage unclarity in various sectors.

2.4.2 VIKOR

Opricovic [60] introduced VIKOR which is regarded as a leading method in MCDM for situations involving multiple and conflicting criteria. Unlike TOPSIS and other distance-based methods, VIKOR looks for solutions that provide a good balance between the group utility and individual regret. Considering both criteria, VIKOR seeks optimal solutions that are also balanced and fair and that benefit the entire group of stakeholders during the decision process. Due to its ability to adapt to various methods and support agreements among experts, VIKOR is widely used in numerous fields such as engineering, environmental management, optimizing supply chains and socio-economic planning.

Given growing uncertainty in decisions, researchers have adopted the VIKOR method with principles of fuzzy set theory to handle data that is vague and lacking information. This fusion has enhanced decision-making in diverse domains. In infrastructure planning, Mahmudah *et al.,* [61] applied fuzzy VIKOR combined with fuzzy AHP to select an optimal nuclear power plant site in Indonesia, focusing on socio-economic factors. In supply chain and organizational contexts, Phan *et al.,* [62] identified key barriers to supply chain resilience in Vietnamese small and medium-size enterprises (SMEs), while Lam *et al.,* [63] ranked the financial performance of Malaysian construction firms using an entropy–fuzzy VIKOR approach. In the education sector, Ayouni *et al.,* [64] evaluated learning management systems in Saudi universities. In tourism and human resources, Hosseini *et al.,* [65] prioritized COVID-19 recovery strategies for ecotourism centers using fuzzy DEMATEL combined with fuzzy VIKOR, while Öztürk and Kaya [66] applied fuzzy VIKOR to support personnel selection in the automotive industry, helping identify the best candidate among several alternatives. These studies demonstrate how fuzzy VIKOR facilitates rational, equitable decisions under uncertainty across varied applications.

To further confront the challenges posed by indeterminacy and conflicting information, scholars have extended the VIKOR method using neutrosophic set theory, offering a richer framework for modeling uncertainty. This enhanced approach has proven effective in varied decision-making scenarios. In legal and cybersecurity domains, Uluçay *et al.,* [67] proposed a VIKOR model based on Q-single-valued neutrosophic sets to evaluate cyber warfare strategies, demonstrating its capacity to reflect expert hesitation and support balanced national policies. In education, Álvarez Enríquez *et al.,* [68] used neutrosophic VIKOR to evaluate child development strategies in Ecuadorian schools. In sustainable supply chain management, Luo *et al.,* [69] used SVNS-VIKOR for supplier selection by integrating entropy and AHP to improve the reliability of weighting criteria. In criminal justice, Paronyan *et al.,* [70] applied the neutrosophic VIKOR method to propose changes to Article 189 of Ecuador's Criminal Code, with the goal of making penalties more proportional to the harm caused. In environmental engineering, Kamal *et al.,* [71] applied SVNS-VIKOR to decide on the best wastewater treatment technologies while considering both subjective and objective aspects. All these studies have shown that neutrosophic VIKOR is effective in helping with decisions that require judgment under high uncertain conditions.

3. Preliminaries

Definition 1. [72] Let X represent a universe or a non-empty set. A Pythagorean neutrosophic set characterized by τ and η as its dependent neutrosophic components is defined as follows:

$$A = \{(x, \tau_A(x), \xi_A(x), \eta_A(x)) | x \in X\} \tag{6}$$

Where τ_A denotes the level of membership, ξ_A denotes the degree of uncertainty, and η_A denotes the extent of non-membership. All the components τ, ξ, η should fall between $[0,1]$, and the following conditions apply:

$$\tau_A(x) + \eta_A(x) \leq 1 \tag{7}$$

$$0 \leq \left(\tau_A(x)\right)^2 + (\eta_A(x))^2 \leq 1 \tag{8}$$

$$0 \leq \left(\tau_A(x)\right)^2 + \left(\xi_A(x)\right)^2 + \left(\eta_A(x)\right)^2 \leq 2 \tag{9}$$

Definition 2. [73] When considering any two PNSs, denoted as $x_1 = \left(\tau_{x_1}, \xi_{x_1}, \eta_{x_1}\right)$, $x_2 = \left(\tau_{x_2}, \xi_{x_2}, \eta_{x_2}\right)$ and $x = (\tau_x, \xi_x, \eta_x)$, the following operational rules apply:

i. $$x_1 \oplus x_2 = \left(\sqrt{{\tau_{x_1}}^2 + {\tau_{x_2}}^2 - {\tau_{x_1}}^2 {\tau_{x_2}}^2}, \xi_{x_1}\xi_{x_2}, \eta_{x_1}\eta_{x_2}\right) \tag{10}$$

ii. $$x_1 \otimes x_2 = \left(\tau_{x_1}\tau_{x_2}, \xi_{x_1} + \xi_{x_2} - \xi_{x_1}\xi_{x_2}\sqrt{{\eta_{x_1}}^2 + {\eta_{x_2}}^2 - {\eta_{x_1}}^2 {\eta_{x_2}}^2}\right) \tag{11}$$

iii. $$\mu x = \left(\sqrt{1 - (1 - {\tau_x}^2)^\mu}, {\xi_x}^\mu, {\eta_x}^\mu\right) \text{ where } \mu \in \Re \text{ and } \mu \geq 0 \tag{12}$$

iv. $$x^\mu = \left({\tau_x}^\mu, 1 - (1 - \xi_x)^\mu, \sqrt{1 - (1 - {\eta_x}^2)^\mu}\right) \text{ where } \mu \in \Re \text{ and } \mu \geq 0 \tag{13}$$

4. New neutrosophic FIQ distance measure

The Flexible Indeterminacy Quantifier (FIQ) brings three key improvements to increase how distance is measured in a neutrosophic set environment: adaptive weighting, self-regulating exponents and dynamic norm selection. The use of adaptive weighting ensures that truth, falsity and indeterminacy all have influence that matches their importance, so none is overpowered in calculating the distance. Self-adjusting exponents help adjusting the distance measure by either highlighting or softening differences based on uncertainty levels. This makes sure that small changes are noticeable when uncertainty is low, but avoids overreacting when uncertainty is high. Dynamic norms make the indicator more flexible by taking actions similar to Manhattan (p≈1) for high uncertainty and then acting like the Euclidean (p≈2) way for low uncertainty. With all these mechanisms, FIQ is highly adaptable and considers both uncertainty and robustness, which makes it ideal for MCDM and complex environments involving ambiguity.

Let $A = (\tau_A, \xi_A, \eta_A)$ and $B = (\tau_B, \xi_B, \eta_B)$ be two neutrosophic sets, where τ, ξ, and η represent the truth-membership, indeterminacy-membership, and falsity-membership degrees, respectively.
The Flexible Indeterminacy Quantifier (FIQ) distance is defined as:

$$d_{FIQ}(A, B) = \frac{1}{3}\sum_{i=1}^{n}\left[w_\tau \cdot (|\tau_A - \tau_B|^{p_\tau}) + w_\xi \cdot (|\xi_A - \xi_B|^{p_\xi}) + w_\eta \cdot (|\eta_A - \eta_B|^{p_\eta})\right]^{\frac{1}{p}} \tag{14}$$

where the terms w_τ, w_ξ and w_η represent adaptive weights that dynamically scale the contribution of each component, and p_τ, p_ξ, p_η and p are power parameters and norm order that regulate the sensitivity of the distance to variations in uncertainty. These are defined as follows:
Adaptive Weights:

$$w_\tau = 1 - \xi_A\xi_B, w_\xi = 1 + \frac{|\tau_A - \eta_B| + |\eta_A - \tau_B|}{2}, w_\eta = 1 - \xi_A\xi_B$$

Power parameters:

$$p_\tau = 1 + \frac{\xi_A + \xi_B}{2}, p_\xi = 2 - |\tau_A - \eta_B|, p_\eta = 1 + \frac{\xi_A + \xi_B}{2}$$

Norm order:

$$p = 2 - \xi_A\xi_B$$

And the normalized FIQ distance between two neutrosophic sets, A and B can be defined as follows:

$$d_{FIQ}^N(A, B) = \frac{1}{3n}\sum_{i=1}^{n}\left[w_\tau \cdot (|\tau_A - \tau_B|^{p_\tau}) + w_\xi \cdot (|\xi_A - \xi_B|^{p_\xi}) + w_\eta \cdot (|\eta_A - \eta_B|^{p_\eta})\right]^{\frac{1}{p}} \tag{15}$$

Here, n denotes the number of criteria.

This formulation allows the distance measure to dynamically adjust based on the levels of indeterminacy and how the elements in a neutrosophic set relate to each other. Thus, this distance measure can adapt to better fit different assessment situations. In the FIQ distance formulation, the term $|\tau_A - \eta_B| + |\eta_A - \tau_B|$ is strategically introduced within the adaptive weight w_ξ and exponent p_ξ to account for the semantic opposition between the truth and falsity components of two PNSs. This expression captures the cross-dimensional conflict by quantifying how far the truth degree of one set deviates from the falsity of the other, and vice versa. If the numbers are large, it means there is more disagreement and higher uncertainty between the sets. As a result, the indeterminacy component is adaptively emphasized by increasing its weight and lowering its exponent, allowing the distance measure to become more sensitive in uncertain decision scenarios. Besides, because the term is symmetrical, the distance will be equal no matter which set comes first under the interchange of A and B, so the symmetry property is always maintained.

To ensure mathematical validity and applicability within a neutrosophic framework, a distance function needs to meet four key properties of a metric space: non-negativity, identity of indiscernibles, symmetry, and the triangle inequality. The proposed FIQ distance is examined against these properties as follows.

(DP1) Non-Negativity

For any two PNSs, A and B, the FIQ distance satisfies:

$$d_{FIQ}(A,B) \geq 0$$

This property is met due to the use of absolute difference operators and non-negative power functions when computing distance. Since all terms are non-negative and aggregated using a generalized p-norm, the distance value is always non-negative.

(DP2) Identity of Indiscernibles

The FIQ distance measure returns a zero value if and only if the two PNSs are identical:

$$d_{FIQ}(A,B) = 0 \Leftrightarrow A = B$$

This condition holds because the absolute value of the difference between the truth, indeterminacy and falsity membership degrees is zero for each component only when the relevant components in sets A and B are equal. Hence, the distance only equals zero if $\tau_A = \tau_B$, $\xi_A = \xi_B$ and $\eta_A = \eta_B$.

(DP3) Symmetry

The FIQ measure is symmetric with respect to the sets being compared:

$$d_{FIQ}(A,B) = d_{FIQ}(B,A)$$

This property is guaranteed by the inherently symmetric structure of the FIQ distance formulation, wherein all membership differences are calculated using absolute values and both the adaptive weights and scaling exponents are derived from expressions that remain invariant under the interchange of sets A and B.

(DP4) Triangle Inequality

To prove the Triangle Inequality, we must show that:

$$d_{FIQ}(A,C) \leq d_{FIQ}(A,B) + d_{FIQ}(B,C)$$

Using the Minkowski inequality:

$$\left(\sum |X_A - X_C|^p\right)^{\frac{1}{p}} \leq \left(\sum |X_A - X_B|^p\right)^{\frac{1}{p}} + \left(\sum |X_B - X_C|^p\right)^{\frac{1}{p}}$$

Since FIQ is based on a Minkowski-like norm with dynamically adjusted exponents, we apply this property component-wise:

For the truth component:

$$\left(w_\tau \cdot (|\tau_A - \tau_C|^{p_\tau})\right)^{\frac{1}{p}} \leq \left(w_\tau \cdot (|\tau_A - \tau_B|^{p_\tau})\right)^{\frac{1}{p}} + \left(w_\tau \cdot (|\tau_B - \tau_C|^{p_\tau})\right)^{\frac{1}{p}}$$

For the indeterminacy component:

$$\left(w_\xi \cdot (|\xi_A - \xi_C|^{p_\xi})\right)^{\frac{1}{p}} \leq \left(w_\xi \cdot (|\xi_A - \xi_B|^{p_\xi})\right)^{\frac{1}{p}} + \left(w_\xi \cdot (|\xi_B - \xi_C|^{p_\xi})\right)^{\frac{1}{p}}$$

For the falsity component:

$$\left(w_\eta \cdot (|\eta_A - \eta_C|^{p_\eta})\right)^{\frac{1}{p}} \leq \left(w_\eta \cdot (|\eta_A - \eta_B|^{p_\eta})\right)^{\frac{1}{p}} + \left(w_\eta \cdot (|\eta_B - \eta_C|^{p_\eta})\right)^{\frac{1}{p}}$$

Since each of the three components independently satisfies the triangle inequality, their weighted Minkowski sum also adheres to this property. Therefore, the FIQ distance satisfies the triangle inequality:

$$d_{FIQ}(A,C) \leq d_{FIQ}(A,B) + d_{FIQ}(B,C)$$

Hence, property (DP4) is fulfilled. This completes the proof.

5. Pythagorean neutrosophic TOPSIS and VIKOR

This section introduces the development of the PNTOPSIS and PNVIKOR methodologies, providing a clear explanation of their structure and key steps. The overall framework and process of the proposed approaches are outlined in detail and visually summarized in Figure 1.

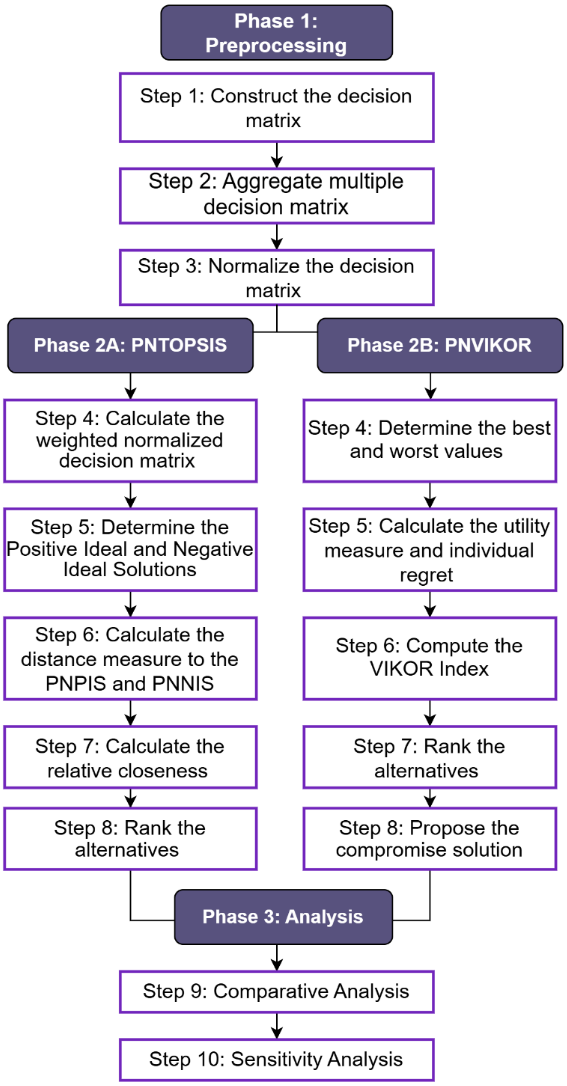

Fig. 1. The PNTOPSIS and PNVIKOR framework

5.1 PNTOPSIS

This section introduces PNTOPSIS, a novel adaptation of the classical TOPSIS technique that maintains its foundational structure while integrating the PNS framework. This enhancement equips the method to effectively capture the uncertainty, indeterminacy, and vagueness inherent in real-world decision-making scenarios. Designed for MCDM problems, PNTOPSIS offers a structured and practical approach for identifying the optimal alternative among multiple options. The methodological steps are outlined as follows:

Step 1. Construct the decision matrix containing scores that demonstrate the ratings or scores of each alternative in relation to each criterion. The rating provided by the decision maker assesses how well each alternative performs in relation to attribute. Following that, each of these scores is transformed into PNS numbers in the form of $x_{ij}^k = \langle \tau_{ij}^k,\ \xi_{ij}^k,\ \eta_{ij}^k \rangle$. The rating scale for PNS numbers relies on the utilization of nine linguistic scores, employing PNS linguistic variables, as elaborated in Table 1. Consider a decision matrix with m alternatives and n criteria, the form will be as follows:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix} \tag{16}$$

Table 1
9-point scale Pythagorean neutrosophic linguistic variable [37]

Score	Linguistic Variable	Rating Scale in PNS
1	Extremely Low Effect	(0.05,0.90,0.95)
2	Very Low Effect	(0.10,0.85,0.90)
3	Low Effect	(0.20,0.80,0.75)
4	Medium Low Effect	(0.35,0.65,0.60)
5	Medium Effect	(0.50,0.50,0.45)
6	Medium High Effect	(0.65,0.35,0.30)
7	High Effect	(0.80,0.25,0.20)
8	Very High Effect	(0.90,0.15,0.10)
9	Extremely High Effect	(0.95,0.05,0.05)

Step 2. Aggregate multiple decision matrix into a single decision matrix. In the process of group decision-making, it is essential to merge all individual evaluations into a collective perspective, resulting in an aggregated neutrosophic decision matrix. There are multiple aggregating operator available for this task. For this study, we will employ the Simple Average method, operating under the assumption that all decision-makers are equally important.

$$X_{ij}^k = \frac{1}{m}\sum_{p=1}^{m}\left(\tau_{ij}^{(p)}, \xi_{ij}^{(p)}, \eta_{ij}^{(p)}\right) \tag{17}$$

Where X_{ij}^k represents the score of the ith alternative on the jth criterion given by the pth decision maker.

Step 3. Normalize the decision matrix (N). Normalization is a process of transforming the various criteria scales into a comparable scale. Through normalization, data is transformed to adhere to a consistent standard. In this scenario, the normalized value is scaled between 0 and 1 [74-75]. In this study, we modified the 0-1 Interval Normalization [76] in the PNS form of $n_{ij}^k = \langle \tau_{ij}^k,\ \xi_{ij}^k,\ \eta_{ij}^k \rangle$.

$$n_{ij} = \begin{cases} \left(\dfrac{\tau_{ij}^k}{\max \tau_{ij}^k}, \dfrac{\xi_{ij}^k}{\max \xi_{ij}^k}, \dfrac{\eta_{ij}^k}{\max \eta_{ij}^k} \right), & for\ benefit\ criteria \\ \left(\dfrac{\min \tau_{ij}^k}{\tau_{ij}^k}, \dfrac{\min \xi_{ij}^k}{\xi_{ij}^k}, \dfrac{\min \eta_{ij}^k}{\eta_{ij}^k} \right), & for\ cost\ criteria \end{cases} \tag{18}$$

Step 4. Calculate the weighted normalized decision matrix (N_W). Each criterion has their respective weightage which can be computed by numerous weighting techniques such as the mean weight, standard deviation, statistical variance procedure, Entropy method, CRiteria Importance Through Inter-criteria Correlation (CRITIC) and their modifications [77-79]. For the simplicity of this study, we employ Mean Weight (MW) method. The mean weight is based on the assumption that each criterion holds equal importance.

$$W_j = \frac{1}{n} \tag{19}$$

Where n is the number of criteria.

Next, each element of the normalized decision matrix is multiplied by its corresponding criteria weight as follows:

$$X_{ij}^w = \left(w_j \cdot \tau_{ij}^k, w_j \cdot \xi_{ij}^k, w_j \cdot \eta_{ij}^k \right) \tag{20}$$

Where w_j is the weight of the jth criteria such that $w_j \geq 0$ for $j = 1,2,\dots,n$ and $\sum_{j=1}^{n} w_j = 1$.

Step 5. Determine the Positive Ideal and Negative Ideal Solutions. In practical decision-making, there are two types of attributes: benefit-type and cost-type. Let K_1 be the set of benefit-type criteria and K_2 the cost-type criteria. V_N^+ is the Pythagorean neutrosophic positive ideal solution (PNPIS) and V_N^- is the Pythagorean neutrosophic negative ideal solution (PNNIS) which both in the form of $V_N = \left(\tau_{ij}^w, \xi_{ij}^w, \eta_{ij}^w \right)$.

$$K_1 = \begin{cases} V_N^+ = \left(max\{\tau_{ij}^w\}, min\{\xi_{ij}^w\}, min\{\eta_{ij}^w\} \right) \\ V_N^- = \left(min\{\tau_{ij}^w\}, max\{\xi_{ij}^w\}, max\{\eta_{ij}^w\} \right) \end{cases} \tag{21}$$

$$K_2 = \begin{cases} V_N^+ = \left(min\{\tau_{ij}^w\}, max\{\xi_{ij}^w\}, max\{\eta_{ij}^w\} \right) \\ V_N^- = \left(max\{\tau_{ij}^w\}, min\{\xi_{ij}^w\}, min\{\eta_{ij}^w\} \right) \end{cases} \tag{22}$$

Step 6. Calculate the distance measure to the PNPIS and PNNIS. For this study, we will use the newly introduced distance measure to calculate the separation measure which is FIQ distance measure. The normalized equation of this measure, as given in Eq. (15), is utilized for the calculation of distance, S_i:

The normalized FIQ distance for PNPIS:

$$S_i^+ = \frac{1}{3n} \sum_{i=1}^{n} \left[w_\tau \cdot \left(\left| \tau_{ij}^w - \tau_j^+ \right|^{p_\tau} \right) + w_\xi \cdot \left(\left| \xi_{ij}^w - \xi_j^+ \right|^{p_\xi} \right) + w_\eta \cdot \left(\left| \eta_{ij}^w - \eta_j^+ \right|^{p_\eta} \right) \right]^{\frac{1}{p}} \tag{23}$$

The normalized FIQ distance for PNNIS:

$$S_i^- = \frac{1}{3n} \sum_{i=1}^{n} \left[w_\tau \cdot \left(\left| \tau_{ij}^w - \tau_j^- \right|^{p_\tau} \right) + w_\xi \cdot \left(\left| \xi_{ij}^w - \xi_j^- \right|^{p_\xi} \right) + w_\eta \cdot \left(\left| \eta_{ij}^w - \eta_j^- \right|^{p_\eta} \right) \right]^{\frac{1}{p}} \tag{24}$$

Step 7. Calculate the relative closeness to the Pythagorean neutrosophic ideal solution. The relative closeness of the ith alternative to the ideal solution is defined as:

$$P_i = \frac{S_i^-}{S_i^+ + S_i^-} \tag{25}$$

Where P_i ranges from 0 to 1. A higher value indicates a closer distance to the positive ideal solution.

Step 8. Rank the alternatives based on their relative closeness. The alternative with the highest P_i is regarded as the optimal choice.

5.2 PNVIKOR

This section presents PNVIKOR, an advanced formulation of the traditional VIKOR method, augmented with the PNS framework. The integration of PNS allows the model to better represent the imprecision and inconsistency of evaluation data, which are often encountered in MCDM contexts. PNVIKOR aims to support the selection of a compromise solution that reflects a balanced response to conflicting criteria. The step-by-step procedure is detailed below:

Steps 1 to 3. The first three steps of PNVIKOR are identical to those of PNTOPSIS, as outlined in Section 5.1. These include constructing the decision matrix, aggregating multiple decision matrices, and normalizing the data. PNVIKOR then proceeds with the following steps:

Step 4. Determine the best and worst values for each criterion. For each criterion, determine the best (f_j^+) and worst (f_j^-) values among all alternatives:

$$f_j^+ = \begin{cases} \left(max(\tau_{ij}), min(\xi_{ij}), min(\eta_{ij})\right), & for\ benefit\ criteria \\ \left(min(\tau_{ij}), max(\xi_{ij}), max(\eta_{ij})\right), & for\ cost\ criteria \end{cases} \tag{26}$$

$$f_j^- = \begin{cases} \left(min(\tau_{ij}), max(\xi_{ij}), max(\eta_{ij})\right), & for\ benefit\ criteria \\ \left(max(\tau_{ij}), min(\xi_{ij}), min(\eta_{ij})\right), & for\ cost\ criteria \end{cases} \tag{27}$$

Step 5. Calculate the utility measure (S_i) and individual regret (R_i). To obtain these values, the FIQ distance measure defined in Eq. (15) is incorporated into the original formulation, resulting in the following expression:

$$S_i = \sum_{j=1}^{m} \left(W_j * \frac{\left[w_\tau \cdot \left(\left|\tau_j^+ - \tau_{ij}\right|^{p_\tau}\right) + w_\xi \cdot \left(\left|\xi_j^+ - \xi_{ij}\right|^{p_\xi}\right) + w_\eta \cdot \left(\left|\eta_j^+ - \eta_{ij}\right|^{p_\eta}\right)\right]^{\frac{1}{p}}}{\left[w_\tau \cdot \left(\left|\tau_j^+ - \tau_j^-\right|^{p_\tau}\right) + w_\xi \cdot \left(\left|\xi_j^+ - \xi_j^-\right|^{p_\xi}\right) + w_\eta \cdot \left(\left|\eta_j^+ - \eta_j^-\right|^{p_\eta}\right)\right]^{\frac{1}{p}}} \right) \tag{28}$$

$$R_i = max_j \left(W_j * \frac{\left[w_\tau \cdot \left(\left|\tau_j^+ - \tau_{ij}\right|^{p_\tau}\right) + w_\xi \cdot \left(\left|\xi_j^+ - \xi_{ij}\right|^{p_\xi}\right) + w_\eta \cdot \left(\left|\eta_j^+ - \eta_{ij}\right|^{p_\eta}\right)\right]^{\frac{1}{p}}}{\left[w_\tau \cdot \left(\left|\tau_j^+ - \tau_j^-\right|^{p_\tau}\right) + w_\xi \cdot \left(\left|\xi_j^+ - \xi_j^-\right|^{p_\xi}\right) + w_\eta \cdot \left(\left|\eta_j^+ - \eta_j^-\right|^{p_\eta}\right)\right]^{\frac{1}{p}}} \right) \tag{29}$$

Step 6. Compute the VIKOR Index, Q_i. To reflect a compromise between group utility and individual regret, the Q_i values is computed as follows:

$$Q_i = v * \frac{S_i - S^*}{S^- - S^*} + (1 - v)\frac{R_i - R^*}{R^- - R^*} \tag{30}$$

Where $S^* = \min S_i$, $S^- = \max S_i$, $R^* = \min R_i$, $R^- = \max R_i$,

$v \in [0,1]$ is the weight of the decision strategy representing the majority rule; typically, $v = 0.5$ is adopted to achieve a balanced compromise.

Step 7. Rank the alternatives. All alternatives are ranked independently based on the values of S_i, R_i, and Q_i in ascending order. The alternative with the lowest Q_i is considered closest to the ideal compromise and is provisionally selected as the top-ranked solution.

Step 8. Propose the compromise solution. The top-ranked alternative based on Q_i value is accepted as the compromise solution if it satisfies both of the following conditions:

Condition 1 - Acceptable advantage.

$$Q(A^2) - Q(A^1) \geq DQ \tag{31}$$

where $DQ = \frac{1}{j-1}$, j is the number of alternatives.

Condition 2 - Acceptable stability in decision-making.

The alternative A^1, ranked first according to Q_i, must also be ranked first by either S_i or R_i. If both conditions are satisfied, the alternative A^1 is accepted as the compromise solution.

i. If only Condition 2 is not satisfied, a set of compromise solutions comprising the two top-ranked alternatives $\{A^1, A^2\}$ is proposed.
ii. If Condition 1 is not satisfied, the compromise set includes the top alternatives A^1, A^2, ..., $A^{(M)}$ where $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A^1) < DQ$ for maximum M. This ensures that the included alternatives are "in closeness" to the best solution, providing a reasonable compromise when the advantage of the top-ranked alternative is not sufficiently distinct.

6. Results and discussion

This section is structured into three interrelated subsections. The first subsection presents a case study on digital supplier selection for a Malaysian manufacturing SME, demonstrating the application of both PNTOPSIS and PNVIKOR techniques in a practical decision-making context. The second subsection provides a detailed comparative study in order to analyze and compare the output of these two approaches in multiple dimensions. Finally, in the third subsection, the sensitivity analysis is done by using several distance measures and weighting scenarios thus determining how robust and responsive the PNTOPSIS and PNVIKOR approaches are to different evaluation scenarios.

6.1 Case Application: Digital Supplier Selection for Manufacturing SMEs

This subsection presents a case study on digital supplier selection for manufacturing SMEs which focus on a mid-sized Malaysian firm that aim to implement an enterprise-grade digital system to enhance their operational capabilities. The company seek to identify the most suitable digital solutions provider that can facilitates its growth, competitiveness, and seamless digital transformation. Choosing the right supplier is a crucial decision that involves multiple factors such as the cost of system integration, the reliability of the vendor and the ability to maintain ongoing operation. These factors make the choice of the supplier is an economically or technology important decision.

Digital solution providers who participated in the preliminary screening were reduced to five participants under the shortlist coded as Supplier S_1 to S_5 based on relevance, past project portfolios, and technical capacity. The evaluation process was structured around four key decision criteria that express strategic, technical and financial issues:

i. C1: System Capability (Benefit) - Assesses how well the system supports business growth through key features, scalability, and tech compatibility.
ii. C2: Vendor Support Quality (Benefit) - Looks at the quality of support, training, and updates provided after installation.
iii. C3: Implementation Cost (Cost) - Covers setup, licensing, and integration costs, aiming to keep expenses low.
iv. C4: Operational Disruption Risk (Cost) - Measures the chance of delays, data issues, or workflow problems during rollout.

The objective is to identify a digital supplier that meets functional and support requirements while ensuring cost efficiency and operational continuity which is a crucial balance for SMEs navigating digital transformation amid uncertainty.

6.1.1 PNTOPSIS Implementation

Step 1. Three domain experts, namely the IT Manager, Operations Lead, and Financial Controller, participated in the evaluation, each independently assessing the suppliers based on defined criteria. These assessments, expressed using a 9-point Likert scale, resulted in three separate decision matrices. The components of these matrices are presented in Tables 2 to 4.

Table 2
The decision matrix provided by Expert 1

	C1	C2	C3	C4
S1	8	7	1	1
S2	8	6	4	5
S3	6	9	3	3
S4	5	7	2	4
S5	8	9	4	3

Table 3
The decision matrix provided by Expert 2

	C1	C2	C3	C4
S1	7	5	2	1
S2	9	6	2	4
S3	5	8	4	5
S4	9	7	1	3
S5	6	6	4	4

Table 4
The decision matrix provided by Expert 3

	C1	C2	C3	C4
S1	8	9	1	4
S2	7	7	2	1
S3	6	8	3	2
S4	7	5	4	5
S5	7	8	3	3

Subsequently, the values in the decision matrices are converted into PNS numbers based on the linguistic scale provided in Table 1, which employs a 9-point PNS linguistic variable. Tables 5 to 7 present the decision matrices with the values expressed in PNS numbers.

Table 5
The decision matrix in PNS numbers provided by Expert 1

	C1	C2	C3	C4
S1	(0.90,0.15,0.10)	(0.80,0.25,0.20)	(0.05,0.90,0.95)	(0.05,0.90,0.95)
S2	(0.90,0.15,0.10)	(0.65,0.35,0.30)	(0.35,0.65,0.60)	(0.50,0.50,0.45)
S3	(0.65,0.35,0.30)	(0.95,0.05,0.05)	(0.20,0.80,0.75)	(0.20,0.80,0.75)
S4	(0.50,0.50,0.45)	(0.80,0.25,0.20)	(0.10,0.85,0.90)	(0.35,0.65,0.60)
S5	(0.90,0.15,0.10)	(0.95,0.05,0.05)	(0.35,0.65,0.60)	(0.20,0.80,0.75)

Table 6
The decision matrix in PNS numbers provided by Expert 2

	C1	C2	C3	C4
S1	(0.80,0.25,0.20)	(0.50,0.50,0.45)	(0.10,0.85,0.90)	(0.05,0.90,0.95)
S2	(0.95,0.05,0.05)	(0.65,0.35,0.30)	(0.10,0.85,0.90)	(0.35,0.65,0.60)
S3	(0.50,0.50,0.45)	(0.90,0.15,0.10)	(0.35,0.65,0.60)	(0.50,0.50,0.45)
S4	(0.95,0.05,0.05)	(0.80,0.25,0.20)	(0.05,0.90,0.95)	(0.20,0.80,0.75)
S5	(0.65,0.35,0.30)	(0.65,0.35,0.30)	(0.35,0.65,0.60)	(0.35,0.65,0.60)

Table 7
The decision matrix in PNS numbers provided by Expert 3

	C1	C2	C3	C4
S1	(0.90,0.15,0.10)	(0.95,0.05,0.05)	(0.05,0.90,0.95)	(0.35,0.65,0.60)
S2	(0.80,0.25,0.20)	(0.80,0.25,0.20)	(0.10,0.85,0.90)	(0.05,0.90,0.95)
S3	(0.65,0.35,0.30)	(0.90,0.15,0.10)	(0.20,0.80,0.75)	(0.10,0.85,0.90)
S4	(0.80,0.25,0.20)	(0.50,0.50,0.45)	(0.35,0.65,0.60)	(0.50,0.50,0.45)
S5	(0.80,0.25,0.20)	(0.90,0.15,0.10)	(0.20,0.80,0.75)	(0.20,0.80,0.75)

Step 2. The multiple decision matrices are aggregated into a single decision matrix by applying Eq. (17). Table 8 presents the resulting aggregated decision matrix. The calculation of the aggregated value for Supplier 1 (S_1) under Criterion 1 (C_1) is illustrated as follows:

$$X_{11} = \frac{1}{3}[(0.90,0.15,0.10) + (0.80,0.25,0.20) + (0.90,0.15,0.10)] = (0.87,0.18,0.13)$$

Table 8
The aggregated decision matrix

	C1	C2	C3	C4
S1	(0.87,0.18,0.13)	(0.75,0.27,0.23)	(0.07,0.88,0.93)	(0.15,0.82,0.83)
S2	(0.88,0.15,0.12)	(0.70,0.32,0.27)	(0.18,0.78,0.80)	(0.30,0.68,0.67)
S3	(0.60,0.40,0.35)	(0.92,0.12,0.08)	(0.25,0.75,0.70)	(0.27,0.72,0.70)
S4	(0.75,0.27,0.23)	(0.70,0.33,0.28)	(0.17,0.80,0.82)	(0.35,0.65,0.60)
S5	(0.78,0.25,0.20)	(0.83,0.18,0.15)	(0.30,0.70,0.65)	(0.25,0.75,0.70)

Step 3. In this step, the aggregated decision matrix is normalized by applying Eq. (18). Table 9 presents the normalized aggregated decision matrix. The following demonstrates the process of calculating the normalized value for Supplier 1 (S_1) with respect to Criterion 1 (C_1).

$$n_{11} = \left(\frac{0.87}{0.88}, \frac{0.18}{0.40}, \frac{0.13}{0.35}\right) = (0.98, 0.46, 0.38)$$

Table 9
The normalized aggregated decision matrix

	C1	C2	C3	C4
S1	(0.98,0.46,0.38)	(0.82,0.80,0.82)	(1.00,0.79,0.70)	(1.00,0.80,0.72)
S2	(1.00,0.38,0.33)	(0.76,0.95,0.94)	(0.36,0.89,0.81)	(0.50,0.95,0.90)
S3	(0.68,1.00,1.00)	(1.00,0.35,0.29)	(0.27,0.93,0.93)	(0.56,0.91,0.86)
S4	(0.85,0.67,0.67)	(0.76,1.00,1.00)	(0.40,0.88,0.80)	(0.43,1.00,1.00)
S5	(0.89,0.63,0.57)	(0.91,0.55,0.53)	(0.22,1.00,1.00)	(0.60,0.87,0.86)

Step 4. Assuming that each criterion holds equal importance, the weight for each criterion is calculated using Eq. (19), resulting in $w = 0.25$. Based on these weights, the weighted normalized aggregated decision matrix is constructed using Eq. (20). Table 10 presents the weighted normalized aggregated decision matrix. The following shows the calculation of the weighted normalized value for Supplier 1 (S_1) under Criterion 1 (C_1).

$$n_{11}^{w} = (0.25 \cdot 0.98, 0.25 \cdot 0.46, 0.25 \cdot 0.38) = (0.25, 0.11, 0.10)$$

Table 10
The weighted normalized aggregated decision matrix

	C1	C2	C3	C4
S1	(0.25,0.11,0.10)	(0.20,0.20,0.21)	(0.25,0.20,0.17)	(0.25,0.20,0.18)
S2	(0.25,0.09,0.08)	(0.19,0.24,0.24)	(0.09,0.22,0.20)	(0.13,0.24,0.23)
S3	(0.17,0.25,0.25)	(0.25,0.09,0.07)	(0.07,0.23,0.23)	(0.14,0.23,0.21)
S4	(0.21,0.17,0.17)	(0.19,0.25,0.25)	(0.10,0.22,0.20)	(0.11,0.25,0.25)
S5	(0.22,0.16,0.14)	(0.23,0.14,0.13)	(0.06,0.25,0.25)	(0.15,0.22,0.21)

Step 5. The PNPIS, V_N^+ and PNNIS, V_N^- values are determined using Eq. (21) and Eq. (22). Table 11 presents the corresponding values.

Table 11
The PNPIS and PNNIS values

	C1	C2	C3	C4
V_N^+	(0.25,0.09,0.08)	(0.25,0.09,0.07)	(0.06,0.25,0.25)	(0.11,0.25,0.25)
V_N^-	(0.17,0.25,0.25)	(0.19,0.25,0.25)	(0.25,0.20,0.17)	(0.25,0.20,0.18)

Step 6. The distance measures to the PNPIS, S_i^+ and PNNIS, S_i^- are calculated using Eq. (23) and Eq. (24). The following illustrates the calculation of the S_i^+ value:
First, the weight and power parameters are calculated.

$$w_\tau = 1 - (0.11)(0.09) = 0.99$$
$$w_\xi = 1 + \frac{|0.25 - 0.08| + |0.10 - 0.25|}{2} = 1.16$$
$$w_\eta = 1 - (0.11)(0.09) = 0.99$$
$$p_\tau = 1 + \frac{0.11 + 0.09}{2} = 1.10$$
$$p_\xi = 2 - |0.25 - 0.08| = 1.84$$
$$p_\eta = 1 + \frac{0.11 + 0.09}{2} = 1.10$$
$$p = 2 - (0.11)(0.09) = 1.99$$

Next, these parameters are inserted into the S^+ formula. The following calculation is specifically for Supplier 1 with respect to Criterion 1.

$$S^+_{(1,1)} = [(0.99)(|0.25 - 0.25|^{1.10}) + (1.16)(|0.11 - 0.09|^{1.83}) + (0.99)(|0.10 - 0.08|^{1.10})]^{\frac{1}{1.99}} = 0.10$$

The values of the remaining parameters for calculating Supplier 1 with respect to Criteria 2, 3, and 4 are provided in Tables 12 and 13. The complete computation of S_i^+ for Supplier 1 is presented below:

$$S_i^+ = \frac{1}{3(4)}\sum_{i=1}^{n} 0.10 + [(0.99)(|0.25 - 0.25|^{1.10}) + (1.16)(|0.11 - 0.09|^{1.84}) + (0.99)(|0.10 - 0.08|^{1.10})]^{\frac{1}{1.99}}$$
$$+ [(0.98)(|0.20 - 0.25|^{1.14}) + (1.09)(|0.20 - 0.09|^{1.87}) + (0.98)(|0.21 - 0.07|^{1.14})]^{\frac{1}{1.98}}$$
$$+ [(0.95)(|0.25 - 0.06|^{1.22}) + (1.06)(|0.20 - 0.25|^{2.00}) + (0.95)(|0.17 - 0.25|^{1.22})]^{\frac{1}{1.95}}$$
$$+ [(0.95)(|0.25 - 0.11|^{1.22}) + (1.04)(|0.20 - 0.25|^{2.00}) + (0.95)(|0.18 - 0.25|^{1.10})]^{\frac{1}{1.95}}$$
$$= 0.1027$$

Table 12
The weight parameters for the calculation of S_i^+

	C1			C2			C3			C4		
	w_τ	w_ξ	w_η	w_τ	w_ξ	w_η	w_τ	w_ξ	w_η	w_τ	w_ξ	w_η
S1	0.99	1.16	0.99	0.98	1.09	0.98	0.95	1.06	0.95	0.95	1.04	0.95
S2	0.99	1.17	0.99	0.98	1.07	0.98	0.94	1.15	0.94	0.94	1.12	0.94
S3	0.98	1.04	0.98	0.99	1.18	0.99	0.94	1.18	0.94	0.94	1.11	0.94
S4	0.98	1.11	0.98	0.98	1.06	0.98	0.95	1.15	0.95	0.94	1.14	0.94
S5	0.99	1.12	0.99	0.99	1.14	0.99	0.94	1.19	0.94	0.95	1.10	0.95

Table 13
The power parameters for the calculation of S_i^+

	C1				C2				C3				C4			
	p_τ	p_ξ	p_η	p	p_τ	p_ξ	p_η	p	p_τ	p_ξ	p_η	p	p_τ	p_ξ	p_η	p
S1	1.10	1.84	1.10	1.99	1.14	1.87	1.14	1.98	1.22	2.00	1.22	1.95	1.22	2.00	1.22	1.95
S2	1.09	1.83	1.09	1.99	1.16	1.88	1.16	1.98	1.24	1.84	1.24	1.94	1.24	1.88	1.24	1.94
S3	1.17	1.91	1.17	1.98	1.09	1.82	1.09	1.99	1.24	1.82	1.24	1.94	1.24	1.89	1.24	1.94
S4	1.13	1.87	1.13	1.98	1.17	1.88	1.17	1.98	1.23	1.85	1.23	1.95	1.25	1.86	1.25	1.94
S5	1.13	1.86	1.13	1.99	1.11	1.85	1.11	1.99	1.25	1.81	1.25	1.94	1.23	1.90	1.23	1.95

The calculation of S_i^- follows the same procedure. Tables 14 and 15 show the parameters used specifically for computing S_i^-, while Table 16 displays the resulting S_i^+ and S_i^- values for each alternative.

Table 14
The weight parameters for the calculation of S_i^-

	C1			C2			C3			C4		
	w_τ	w_ξ	w_η	w_τ	w_ξ	w_η	w_τ	w_ξ	w_η	w_τ	w_ξ	w_η
S1	0.97	1.04	0.97	0.95	1.03	0.95	0.96	1.08	0.96	0.96	1.07	0.96
S2	0.98	1.04	0.98	0.94	1.05	0.94	0.96	1.07	0.96	0.95	1.04	0.95
S3	0.94	1.08	0.94	0.98	1.06	0.98	0.95	1.06	0.95	0.95	1.04	0.95
S4	0.96	1.02	0.96	0.94	1.06	0.94	0.96	1.06	0.96	0.95	1.04	0.95
S5	0.96	1.03	0.96	0.97	1.04	0.97	0.95	1.06	0.95	0.96	1.03	0.96

Table 15
The power parameters for the calculation of S_i^-

	C1				C2				C3				C4			
	p_τ	p_ξ	p_η	p	p_τ	p_ξ	p_η	p	p_τ	p_ξ	p_η	p	p_τ	p_ξ	p_η	p
S1	1.18	2.00	1.18	1.97	1.23	1.95	1.23	1.95	1.20	1.92	1.20	1.96	1.20	1.93	1.20	1.96
S2	1.17	2.00	1.17	1.98	1.24	1.94	1.24	1.94	1.21	1.92	1.21	1.96	1.22	1.95	1.22	1.95
S3	1.25	1.92	1.25	1.94	1.17	2.00	1.17	1.98	1.22	1.89	1.22	1.95	1.21	1.96	1.21	1.95
S4	1.21	1.96	1.21	1.96	1.25	1.94	1.25	1.94	1.21	1.93	1.21	1.96	1.22	1.93	1.22	1.95
S5	1.20	1.97	1.20	1.96	1.19	1.98	1.19	1.97	1.22	1.88	1.22	1.95	1.21	1.97	1.21	1.96

Table 16
The S_i^+ and S_i^- values for each alternative

	S_i^+	S_i^-
S1	0.1027	0.0476
S2	0.0609	0.0954
S3	0.0587	0.0913
S4	0.0790	0.0782
S5	0.0568	0.1090

Step 7. The relative closeness P_i to the Pythagorean neutrosophic ideal solution is determined using Eq. (25), and the P_i values for each alternative are presented in Table 17. The following demonstrates the calculation of the P_i value for Supplier 1 (S_1).

$$P_1 = \frac{0.0476}{0.1027 + 0.0476} = 0.3167$$

Table 17
The P_i values for each alternative

	P_i
S1	0.3167
S2	0.6104
S3	0.6087
S4	0.4973
S5	0.6573

Step 8. The alternatives are ranked according to their relative closeness values, ordered from the highest P_i to the lowest. Table 18 presents the ranking outcomes.

Table 18
The ranking of alternatives based on P_i values

	P_i	Rank
S1	0.3167	5
S2	0.6104	2
S3	0.6087	3
S4	0.4973	4
S5	0.6573	1

6.1.2 PNVIKOR Implementation

Steps 1 to 3. The initial three steps of the PNVIKOR illustrative example follow the same procedure as those in PNTOPSIS, as described in Section 6.1.1.

Step 4. The best and worst values, f_j^+ and f_j^- respectively, for each criterion are determined using Eq. (26) and Eq. (27). Table 19 summarizes the values.

Table 19
The best and worst values

	C1	C2	C3	C4
f_j^+	(1.00,0.38,0.33)	(1.00,0.35,0.29)	(0.22,1.00,1.00)	(0.43,1.00,1.00)
f_j^-	(0.68,1.00,1.00)	(0.76,1.00,1.00)	(1.00,0.79,0.70)	(1.00,0.80,0.72)

Step 5. Calculate the utility measure (S_i) and individual regret (R_i). The utility measure, S_i and the individual regret, R_i are calculated using Eq. (28) and Eq. (29). The weights for each criterion, w_j are consistent with those used in the PNTOPSIS method, with $w = 0.25$. The following demonstrates the computation of the S_i value:

First, the weight and power parameters are calculated. To distinguish between the components used in the two parts of Eq. (18), we define the numerator-specific weights and exponents as $w_{\tau,num}$, $w_{\xi,num}$, $w_{\eta,num}$, $p_{\tau,num}$, $p_{\xi,num}$, $p_{\eta,num}$, p_{num} and the denominator-specific weights and exponents as $w_{\tau,den}$, $w_{\xi,den}$, $w_{\eta,den}$, $p_{\tau,den}$, $p_{\xi,den}$, $p_{\eta,den}$, p_{den} respectively.

$$w_{\tau,num} = 1 - (0.38)(0.46) = 0.83$$
$$w_{\xi,num} = 1 + \frac{|1.00 - 0.38| + |0.33 - 0.98|}{2} = 1.63$$
$$w_{\eta,num} = 1 - (0.38)(0.46) = 0.83$$
$$p_{\tau,num} = 1 + \frac{0.38 + 0.46}{2} = 1.42$$
$$p_{\xi,num} = 2 - |1.00 - 0.38| = 1.38$$
$$p_{\eta,num} = 1 + \frac{0.38 + 0.46}{2} = 1.42$$
$$p_{num} = 2 - (0.38)(0.46) = 1.83$$
$$w_{\tau,den} = 1 - (0.38)(1.00) = 0.62$$
$$w_{\xi,den} = 1 + \frac{|1.00 - 1.00| + |0.33 - 0.68|}{2} = 1.17$$
$$w_{\eta,den} = 1 - (0.38)(1.00) = 0.62$$
$$p_{\tau,den} = 1 + \frac{0.38 + 1.00}{2} = 1.69$$
$$p_{\xi,den} = 2 - |1.00 - 1.00| = 2.00$$
$$p_{\eta,den} = 1 + \frac{0.38 + 1.00}{2} = 1.69$$
$$p_{den} = 2 - (0.38)(1.00) = 1.63$$

Next, the calculated parameters are substituted into the S_i formula. The following computation specifically pertains to Supplier 1 in relation to Criterion 1.

$$S_{(1,1)} = (0.25)\left(\frac{[(0.83)(|1.00 - 0.98|^{1.42}) + (1.63)(|0.38 - 0.46|^{1.38}) + (0.83)(|0.33 - 0.38|^{1.42})]^{\frac{1}{1.83}}}{[(0.62)(|1.00 - 0.68|^{1.69}) + (1.17)(|0.38 - 1.00|^{2.00}) + (0.62)(|0.33 - 1.00|^{1.69})]^{\frac{1}{1.63}}}\right)$$
$$= 0.06$$

The parameter values required to calculate Supplier 1 with respect to Criteria 2, 3, and 4 are summarized in Tables 20, 21, 22, and 23. Table 24 presents the computed S_i and R_i values. A comprehensive demonstration of the S_i computation for Supplier 1 is provided below.

$$S_i = 0.06 + (0.25)\left(\frac{[(0.72)(|1.00-0.82|^{1.58}) + (1.35)(|0.35-0.80|^{1.82}) + (0.72)(|0.29-0.82|^{1.58})]^{\frac{1}{1.72}}}{[(0.65)(|1.00-0.76|^{1.68}) + (1.23)(|0.35-1.00|^{2.00}) + (0.65)(|0.29-1.00|^{1.68})]^{\frac{1}{1.65}}}\right)$$

$$+ (0.25)\left(\frac{[(0.21)(|0.22-1.00|^{1.90}) + (1.24)(|1.00-0.79|^{1.53}) + (0.21)(|1.00-0.70|^{1.90})]^{\frac{1}{1.21}}}{[(0.21)(|0.22-1.00|^{1.90}) + (1.24)(|1.00-0.79|^{1.53}) + (0.21)(|1.00-0.70|^{1.90})]^{\frac{1}{1.21}}}\right)$$

$$+ (0.25)\left(\frac{[(0.20)(|0.43-1.00|^{1.90}) + (1.15)(|1.00-0.80|^{1.71}) + (0.20)(|1.00-0.72|^{1.90})]^{\frac{1}{1.20}}}{[(0.20)(|0.43-1.00|^{1.90}) + (1.15)(|1.00-0.80|^{1.71}) + (0.20)(|1.00-0.72|^{1.90})]^{\frac{1}{1.20}}}\right)$$

$$= 0.7600$$

Table 20
The weight parameters utilized in the numerator terms for computing S_i

	C1			C2			C3			C4		
	$w_{\tau,num}$	$w_{\xi,num}$	$w_{\eta,num}$	$w_{\tau,num}$	$w_{\xi,num}$	$w_{\eta,num}$	$w_{\tau,num}$	$w_{\xi,num}$	$w_{\eta,num}$	$w_{\tau,num}$	$w_{\xi,num}$	$w_{\eta,num}$
S1	0.83	1.63	0.83	0.72	1.35	0.72	0.21	1.24	0.21	0.20	1.15	0.20
S2	0.86	1.67	0.86	0.67	1.26	0.67	0.11	1.61	0.11	0.05	1.49	0.05
S3	0.63	1.17	0.63	0.88	1.71	0.88	0.07	1.72	0.07	0.09	1.43	0.09
S4	0.75	1.42	0.75	0.65	1.23	0.65	0.13	1.59	0.13	0.00	1.57	0.00
S5	0.77	1.49	0.77	0.81	1.54	0.81	0.00	1.78	0.00	0.13	1.41	0.13

Table 21
The power parameters utilized in the numerator terms for computing S_i

	C1				C2				C3				C4			
	$p_{\tau,nun}$	$p_{\xi,nun}$	$p_{\eta,nun}$	p_{num}	$p_{\tau,nun}$	$p_{\xi,nun}$	$p_{\eta,nun}$	p_{num}	$p_{\tau,nun}$	$p_{\xi,nun}$	$p_{\eta,nun}$	p_{num}	$p_{\tau,nun}$	$p_{\xi,nun}$	$p_{\eta,nun}$	p_{num}
S1	1.42	1.38	1.42	1.83	1.58	1.82	1.58	1.72	1.90	1.53	1.90	1.21	1.90	1.71	1.90	1.20
S2	1.38	1.33	1.38	1.86	1.65	1.94	1.65	1.67	1.95	1.41	1.95	1.11	1.98	1.53	1.98	1.05
S3	1.69	2.00	1.69	1.63	1.35	1.29	1.35	1.88	1.97	1.29	1.97	1.07	1.95	1.57	1.95	1.09
S4	1.52	1.67	1.52	1.75	1.68	2.00	1.68	1.65	1.94	1.43	1.94	1.13	2.00	1.43	2.00	1.00
S5	1.50	1.57	1.50	1.77	1.45	1.53	1.45	1.81	2.00	1.22	2.00	1.00	1.93	1.57	1.93	1.13

Table 22
The weight parameters utilized in the denominator terms for computing S_i

C1			C2			C3			C4		
$w_{\tau,den}$	$w_{\xi,den}$	$w_{\eta,den}$	$w_{\tau,den}$	$w_{\xi,den}$	$w_{\eta,den}$	$w_{\tau,den}$	$w_{\xi,den}$	$w_{\eta,den}$	$w_{\tau,den}$	$w_{\xi,den}$	$w_{\eta,den}$
0.63	1.17	0.63	0.65	1.23	0.65	0.21	1.24	0.21	0.20	1.15	0.20

Table 23
The power parameters utilized in the denominator terms for computing S_i

C1				C2				C3				C4			
$p_{\tau,den}$	$p_{\xi,den}$	$p_{\eta,den}$	p_{den}	$p_{\tau,den}$	$p_{\xi,den}$	$p_{\eta,den}$	p_{den}	$p_{\tau,den}$	$p_{\xi,den}$	$p_{\eta,den}$	p_{den}	$p_{\tau,den}$	$p_{\xi,den}$	$p_{\eta,den}$	p_{den}
1.69	2.00	1.69	1.63	1.68	2.00	1.68	1.65	1.90	1.53	1.90	1.21	1.90	1.71	1.90	1.20

Table 24
The S_i and R_i values for each alternative

	S_i	R_i
S1	0.7600	0.2500
S2	0.3298	0.2361
S3	0.3540	0.2500
S4	0.4945	0.2500
S5	0.3598	0.1347

Step 6. Using Eq. (30), the VIKOR index Q_i is computed for each alternative. The results are summarized in Table 25. The following illustrates the computation of the Q_i value for Supplier 1 (S_1):

$$Q_1 = (0.5)\left(\frac{0.7600 - 0.3298}{0.7600 - 0.3298}\right) + (1 - 0.5)\left(\frac{0.2500 - 0.1347}{0.2500 - 0.1347}\right) = 1.0000$$

Table 25
The Q_i values for each alternative

	Q_i
S1	1.0000
S2	0.4398
S3	0.5281
S4	0.6913
S5	0.0348

Step 7. The alternatives are ranked independently according to their Q_i values, from the lowest to the highest. The alternative with the lowest Q_i represents the ideal compromise solution. Table 26 displays the ranking results.

Table 26
The ranking of alternatives based on VIKOR Index

	Q_i	Rank
S1	1.0000	5
S2	0.4398	2
S3	0.5281	3
S4	0.6913	4
S5	0.0348	1

Step 8. The compromise solution for the PNVIKOR method was determined based on the three core measures: S_i (group utility), R_i (individual regret), and Q_i (composite index). As shown in Table 18, Supplier 5 (S_5) emerged as the best compromise solution, attaining the lowest value of $Q_i = 0.0348$, which is significantly better than the second-best supplier, S_2 ($Q_i = 0.4398$). This is a strong indicator of its superiority across the decision criteria.

To validate the robustness of this solution, two compromise conditions of the PNVIKOR method must be examined:

Condition 1 - Acceptable Advantage:

$$Q(A^2) - Q(A^1) \geq DQ$$

$$Q(S_2) - Q(S_5) = 0.4398 - 0.0348 = 0.4050 \geq \frac{1}{5-1} = 0.25$$

Thus, Condition 1 is satisfied.

Condition 2 - Acceptable stability in decision making:

Supplier 5 (S_5) should also be ranked the best in either S_i or R_i. In this case, S_5 has the lowest $R_i = 0.1347$, indicating minimal individual regret across all criteria. Therefore, Condition 2 is also satisfied.

Since both conditions are fulfilled, Supplier 5 (S_5) is accepted as the unique compromise solution. The dominance of S_5 is reinforced by its balanced performance in both group satisfaction and individual regret measures, indicating that it not only offers collective benefit but also minimizes dissatisfaction among stakeholders.

6.2 Comparative Analysis

This section presents a comparative evaluation of the PNTOPSIS and PNVIKOR methods, focusing on their effectiveness in ranking decision alternatives. Table 27 displays the ranking outcomes of PNTOPSIS and PNVIKOR.

Table 27
The PNTOPSIS and PNVIKOR result comparison

	PNTOPSIS	Rank	PNVIKOR	Rank
S1	0.3167	5	1.0000	5
S2	0.6104	2	0.4398	2
S3	0.6087	3	0.5281	3
S4	0.4973	4	0.6913	4
S5	0.6573	1	0.0348	1

The comparative analysis between the PNTOPSIS and PNVIKOR methods reveals consistent rankings and insightful methodological contrasts within the Pythagorean neutrosophic decision-making framework. Both methods unanimously identify the best and worst-performing alternatives. Supplier 5 (S_5) is consistently ranked first, while Supplier 1 (S_1) holds the lowest rank in both approaches. This strong alignment underscores the robustness of these alternatives under different computational logics. Specifically, S_5 demonstrates high closeness to the ideal solution in PNTOPSIS (score: 0.6573) and minimal regret in PNVIKOR (score: 0.0348), whereas S_1 exhibits the weakest performance across all criteria.

Beyond the extremes, alternatives S_2, S_3, and S_4 also retain their respective rankings, namely second, third, and fourth, in both methods. Despite slight variations in their numerical scores, with PNTOPSIS offering a tighter range (0.3167 to 0.6573) and PNVIKOR a more dispersed one (0.0348 to 1.0000), the preservation of rank order indicates stable mid-tier performance. The absence of ranking conflicts suggests that the decision-making criteria are well-balanced and that the two methods complement each other in identifying the optimal choice.

However, a deeper examination of the score distributions reveals meaningful methodological contrasts between the two decision-making approaches. PNTOPSIS calculates the relative closeness of each alternative to the ideal and negative-ideal solutions using geometric distances, resulting in proportionally scaled scores. This design helps ensure balanced evaluations by focusing on how close each option comes to the optimal outcome. On the other hand, PNVIKOR includes group utility and individual regret under its scope and favors compromising arrangements. This formulation causes it to highlight flaws in an alternative's performance by penalizing alternatives with poor outcomes in any single criterion. This can be seen in the more pronounced score separation between S_5 and the other alternatives under PNVIKOR than the smoother separation in PNTOPSIS. Theoretically, these methods reflect distinct philosophies where PNTOPSIS supports holistic decision-making advantages designed for those that prefer balanced performance, while PNVIKOR is better suited for cases that need fairness and want to mitigate dissatisfaction in every criterion. While both methods produce the same rankings in this analysis, the way they score reveals subtle differences in how performance is interpreted.

To validate these conceptual distinctions, a statistical comparison was carried out to test whether the outcomes matched one another. To be specific, the Pearson correlation coefficient between the results of PNTOPSIS and PNVIKOR was calculated, yielding a value of –0.9100. The strong negative correlation shows a steady inverse relationship between the two methods, which makes sense given their scoring styles. PNTOPSIS gives higher scores to better alternatives, while PNVIKOR gives lower scores. The negative sign does not indicate disagreement but it simply reflects that the two methods rank performance in opposite directions. If the correlation is close to zero, it would suggest that the rankings are inconsistent. On the other hand, a strong positive correlation would go against the core idea behind PNVIKOR. Therefore, this near-perfect inverse relationship shows that, even though the two methods use different calculations, they still arrive at the same decision outcome. The fact that both methods lead to the same outcome adds to the reliability of the results, highlighting how PNTOPSIS and PNVIKOR complement each other in assessing the alternatives from different angles.

6.3 Sensitivity Analysis

This section presents a sensitivity analysis to evaluate the reliability and robustness of the PNTOPSIS and PNVIKOR methodologies. The analysis is based on the results obtained by using three different distance measures: the FIQ distance, PN-Hamming distance and PN-Euclidean distance. In addition to varying the distance metrics, the analysis also incorporates five distinct weightage scenarios to reflect different decision-making priorities, including equal importance, quality-focused, cost-conscious, balanced, and logistics-sensitive preferences. Tables 28 and 29 present the results for PNTOPSIS and PNVIKOR using the selected distance metrics. To make the comparison easier to understand, the rankings are also shown visually in Figures 2 and 3. This evaluation helps confirm the stability of the results and the robustness of the methods with different performance indicators.

Table 28 and Figure 2 provide the outcomes obtained when using FIQ, PN-Hamming, and PN-Euclidean distance in the PNTOPSIS model. All three measures give the same outcomes, where alternative S_5 is chosen as the most preferred and S_2 is second, followed by S_3, S_4, and S_1. Having a similar ranking with different distances reveals that the decision-making process is reliable and not affected by the distance itself.

Even though the rankings do not change, there are notable differences in the computed relative closeness values. Values generated by the FIQ distance lie between 0.3167 and 0.6573 which shows the distribution is fairly compact. PN-Hamming shows the widest score range, from 0.3412 to 0.7161, which indicates a clearer distinction between the best and worst alternatives. In contrast, PN-Euclidean has the smallest range, from 0.4122 to 0.7053 due to its use of squaring tends to reduce noticeable differences. Because of this squaring, the impact of unusual deviations is tamed and the values are compressed. Since PN-Hamming has broader gap, it partly exaggerates differences, while FIQ presents a more nuanced and balanced comparison of the concepts. The main feature of FIQ is that it can adjust its weighting and non-linear scaling that adjust based on the level of indeterminacy in the data. This makes it especially effective in situations where information is not certain. In contrast, PN-Hamming and PN-Euclidean treat truth, indeterminacy, and falsity in a fixed and uniform way, leading to a more rigid and straightforward evaluation.

Table 28
The performance of PNTOPSIS method using FIQ, PN-Hamming, and PN-Euclidean distances

	FIQ	PN-Hamming	PN-Euclidean
S1	0.3167	0.3412	0.4122
S2	0.6104	0.6145	0.5748
S3	0.6087	0.6099	0.5741
S4	0.4973	0.4819	0.4777
S5	0.6573	0.7161	0.7053

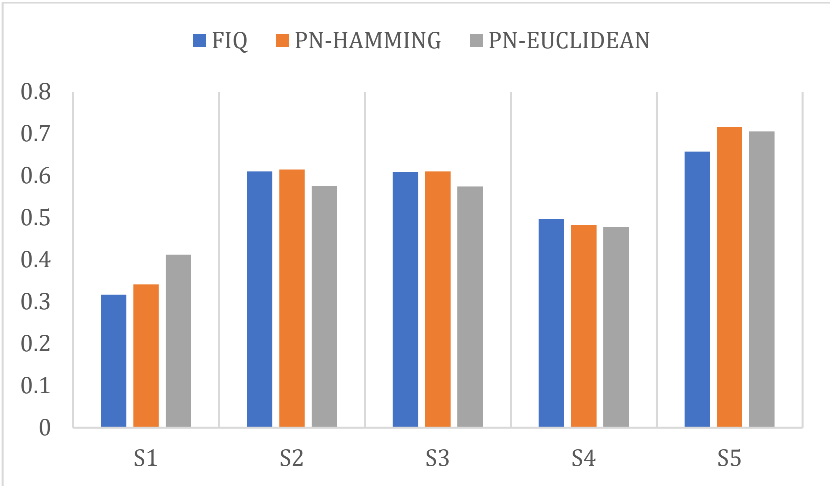

Fig. 2. Comparison of PNTOPSIS rankings across three distance measures

Table 29
The performance of PNVIKOR method using FIQ, PN-Hamming, and PN-Euclidean distances

	FIQ	PN-Hamming	PN-Euclidean
S1	1.0000	1.0000	1.0000
S2	0.4398	0.5652	0.6017
S3	0.5281	0.6291	0.7222
S4	0.6913	0.7486	0.7767
S5	0.0348	0.0000	0.0000

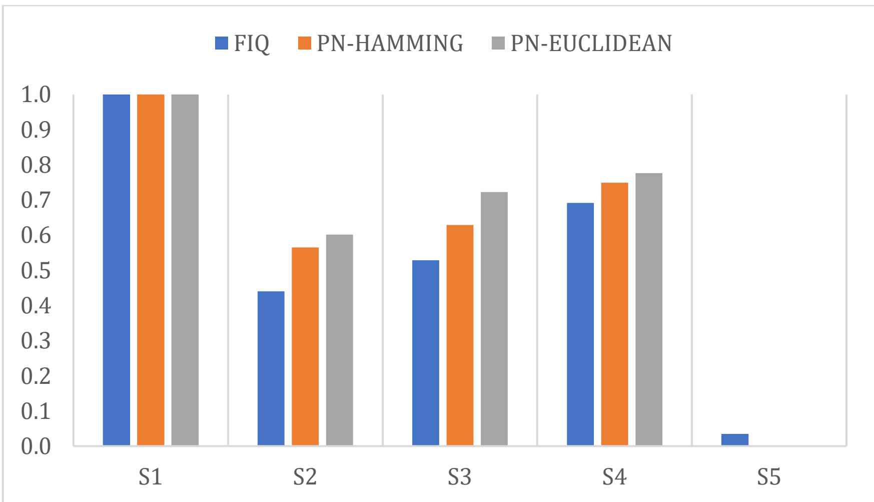

Fig. 3. Comparison of PNVIKOR rankings across three distance measures

Table 29 and Figure 3 summarize the application of the same three distance measures in the PNVIKOR method. Once again, the rankings are identical for all methods, with S_5 ranked as the best alternative, followed by S_2, S_3, S_4, and S_1. This reaffirms the stability of the decision-making framework across different distance functions.

Despite uniform rankings, the distance values vary slightly. FIQ produces values from 0.0348 to 1.0000, while PN-Hamming ranges from 0.0000 to 1.0000, and PN-Euclidean from 0.0000 to 1.0000 as well. In all methods, S_5 achieves the lowest score, making it the ideal alternative in all cases, while S_1 registers the highest score, confirming it as the worst-performing alternative.

Overall, even though all three distance measures produce the same ranking, their numerical distance value differ slightly due to their internal calculation of the distances. Since the FIQ method is adaptable, it might be preferred in complex and indeterminate environments. On the other hand, PN-Hamming typically provide clearer separation between alternatives because of its linear sensitivity to differences in each component. This can be especially useful in situations where precise distinctions are needed. Meanwhile, although PN-Euclidean is more conservative in capturing variations, it still produces consistent ranking results, which might be a better choice in situations where it is important to reduce the influence of outliers.

Next, to make sure the framework works well in different situations, five different weighting scenarios were tested. Each scenario was carefully created to reflect how different stakeholders might see the problem and what they might value most. These scenarios are explained as follows:

i. C1: Equal weighting across all criteria
ii. C2: Emphasis on product quality and variety
iii. C3: Focus on cost efficiency and pricing sensitivity
iv. C4: Balanced trade-off with slight preference for quality attributes
v. C5: Logistics and delivery cost-focused prioritization

Table 30 provides an overview of the criteria weight sets used in the sensitivity analysis, each one reflecting a different decision-making viewpoint.

Tables 31 and 32 display the outcomes of PNTOPSIS and PNVIKOR under five distinct weight configurations. For clearer comparison, these results are also illustrated in Figures 4 and 5.

As shown in Table 31 and Figure 4, the sensitivity analysis of the PNTOPSIS method under five different weighting scenarios shows a high degree of ranking stability. Regardless of which criteria are emphasized, Supplier 5 (S_5) consistently emerges as the top performer, making it a clear front-runner across all scenarios. This consistent success suggests that S_5 is a strong all-around option that holds up well even when stakeholder priorities shift. On the other hand, Supplier 1 (S_1) consistently ranks at the bottom, pointing to overall weak performance across the evaluated dimensions. Sometimes S_2 takes second place and S_3 takes third, but S_2 usually does better in situations where benefits are important and S_3 is a bit better when aiming for low costs. Supplier 4 (S_4) consistently stays in the fourth position, indicating a stable yet unremarkable performance that falls short of competing with the leading suppliers.

Table 30
The weights used in each scenario

	w_1	w_2	w_3	w_4
C1	0.25	0.25	0.25	0.25
C2	0.40	0.30	0.15	0.15
C3	0.15	0.15	0.35	0.35
C4	0.35	0.25	0.20	0.20
C5	0.20	0.20	0.25	0.35

These outcomes underscore the robustness and discriminatory strength of the PNTOPSIS method, as its ranking results remain consistent despite changes in weight configurations. This consistency makes it highly suitable for practical decision making where stability in preference is essential.

Table 31
The PNTOPSIS results under five different weight scenarios

	C1	C2	C3	C4	C5
S1	0.3167	0.3639	0.2808	0.3471	0.3019
S2	0.6104	0.6069	0.6351	0.6195	0.6231
S3	0.6087	0.5631	0.6363	0.5748	0.6190
S4	0.4973	0.4564	0.5449	0.4825	0.5208
S5	0.6573	0.6330	0.6810	0.6451	0.6653

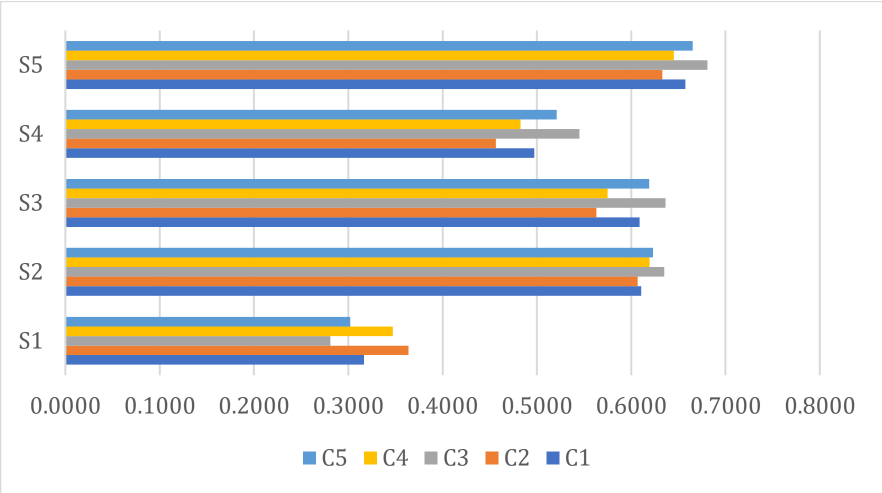

Fig. 4. Comparison of PNTOPSIS scores across weight scenarios

Table 32
The PNVIKOR results under five different weight scenarios

	C1	C2	C3	C4	C5
S1	1.0000	0.5590	1.0000	0.5352	1.0000
S2	0.4398	0.1837	0.0000	0.1471	0.1081
S3	0.5281	0.7065	0.0395	0.6633	0.1695
S4	0.6913	0.6677	0.1025	0.4935	0.2537
S5	0.0348	0.1415	0.0286	0.1096	0.0568

In contrast, Table 32 and Figure 5 show that the PNVIKOR method is more sensitive to changes in criterion weights, leading to noticeable shifts in how the alternatives are ranked. While Supplier 5 (S_5) generally performs well, it does not consistently hold the top position under every weighting scenario. Suppliers like S_2, S_3, and S_4 frequently trade places, depending on which criteria are given more emphasis. Supplier 1 (S_1), which consistently finishes at the bottom in the PNTOPSIS results, shows a more mixed performance in PNVIKOR. While it still ranks last in several scenarios like C_1, C_3, and C_5, it manages to perform slightly better in others, such as C_2 and C_4, where it edges out at least one other supplier.

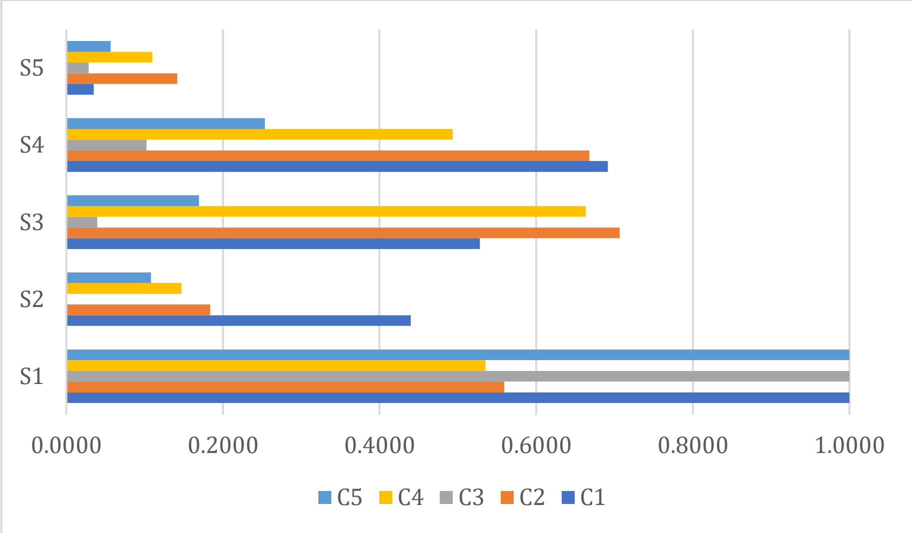

Fig. 5. Comparison of PNVIKOR scores across weight scenarios

This indicates that while S_1 generally struggles to compete, it can occasionally benefit from specific weighting conditions in the PNVIKOR method, allowing it to perform slightly better when certain criteria are given more importance. This variability is mainly caused by the PNVIKOR's compromise-based approach, which balances two key aspects: group utility (S_i) and individual regret (R_i). The final ranking is determined by the Q_i value, which combines both measures according to each scenario assumed weightings. As a result, even slight changes in the importance of certain criteria can shift the balance between utility and regret. This produces noticeable changes in the Q_i values and sometimes even altering the overall rankings. PNVIKOR relies on the best and worst performance values as benchmarks during normalization that makes it becomes particularly reactive when supplier scores are closely matched and makes the method more sensitive to weight changes. Overall, this characteristic makes PNVIKOR a strong choice when fairness and balanced compromise across all criteria are central to the decision-making process.

Therefore, these methods uphold their critical differences: PNTOPSIS works best where dominance matters, while PNVIKOR is more adaptable and able to capture the effects of different stakeholder priorities. This comparison emphasizes the value of choosing a method that aligns with the specific goals of the decision-making process. If the aim is to maintain consistency and overall dominance, PNTOPSIS is the better choice. However, when one prefers a more balanced and compromise focused evaluation that reflects diverse priorities, PNVIKOR proves to be the more suitable choice.

7. Conclusions

This study proposed a decision-making framework for digital supplier selection in manufacturing SMEs which integrates Pythagorean neutrosophic set (PNS) with TOPSIS and VIKOR and further enhanced by a novel Flexible Indeterminacy Quantifier (FIQ) distance measure. The framework was applied to a case study involving the evaluation of multiple digital solution providers thus capturing the complexity and uncertainty that often characterize SME digitalization decisions.

As indicated in the case study, the PNTOPSIS and PNVIKOR approaches produced unified and solid results. The two strategies were close when it came to finding the topmost as well as the worst-ranking suppliers confirming the usefulness of the suggested decision-making model in resolving conflicting criteria in a state of uncertainty. The best overall provider of digital solutions was Supplier

S_5, who provided the optimal technical prowess, stable product support, cost-effectiveness, and lack of integration difficulty and risk of operational chaos.

One important feature of the study was the use of the FIQ distance. It changes its weights and scaling based on the level of indeterminacy in expert input. Compared to traditional distance measures, FIQ offered a more refined way to capture hesitation and uncertainty in evaluations. This results in a smoother score distributions and clearer differences between alternatives. These qualities are valuable in decision-making situation that involves subjective judgment. Sensitivity analysis shows that PNTOPSIS remain stable when weight changed, while PNVIKOR respond well to shifting priority. Together, these methods and the FIQ distance provides a solid approach for both fixed and flexible decision-making need.

In conclusion, the proposed framework support more informed and transparent digital supplier selection in manufacturing SMEs. It upgrades decision quality in context where uncertainty is significant, and lay the groundwork for future development involving more dynamic and data-driven evaluation tool. In the future, this framework could be improved by using objective weighting methods and real-time analytics. This would make it more useful for both long-term strategic planning and everyday operational decisions. These kinds of advancements can help strengthen the theoretical base of neutrosophic MCDM and make it more practical for real-world use. By doing so, it can better support decisions that are not only strategic but also fair and sustainable, especially in the fields of economics and management.

Author Contributions

Conceptualization, M.S.M.K. and Z.M.R.; methodology, M.S.M.K. and Z.M.R.; formal analysis, M.S.M.K.; investigation, M.S.M.K.; resources, Z.M.R.; data curation, M.S.M.K.; writing—original draft preparation, M.S.M.K.; writing—review and editing, Z.M.R., Z.F.Z., N.H.K., N.A., S.A.R. and F.A.; visualization, M.S.M.K.; supervision, Z.M.R.; project administration, Z.M.R.; funding acquisition, Z.M.R. All authors have read and agreed to the published version of the manuscript.

Funding

The publication of this research was supported by funding from the Malaysia Italy Design Institute, Universiti Kuala Lumpur, and by Universiti Teknologi MARA and the Institute of Postgraduate Studies, UiTM.

Data Availability Statement

All data generated or analyzed during this study are included in this published article.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

We would like to acknowledge the Ministry of Higher Education Malaysia for their sponsorship of the Fundamental Research Grant Scheme (Project Code: FRGS/1/2023/STG06/UITM/02/5), which has been instrumental in supporting the research activities presented in this work. We also extend our gratitude to the Malaysia Italy Design Institute, Universiti Kuala Lumpur, Universiti Teknologi MARA, and the Institute of Postgraduate Studies, UiTM, for their financial support in funding the publication of this article.

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